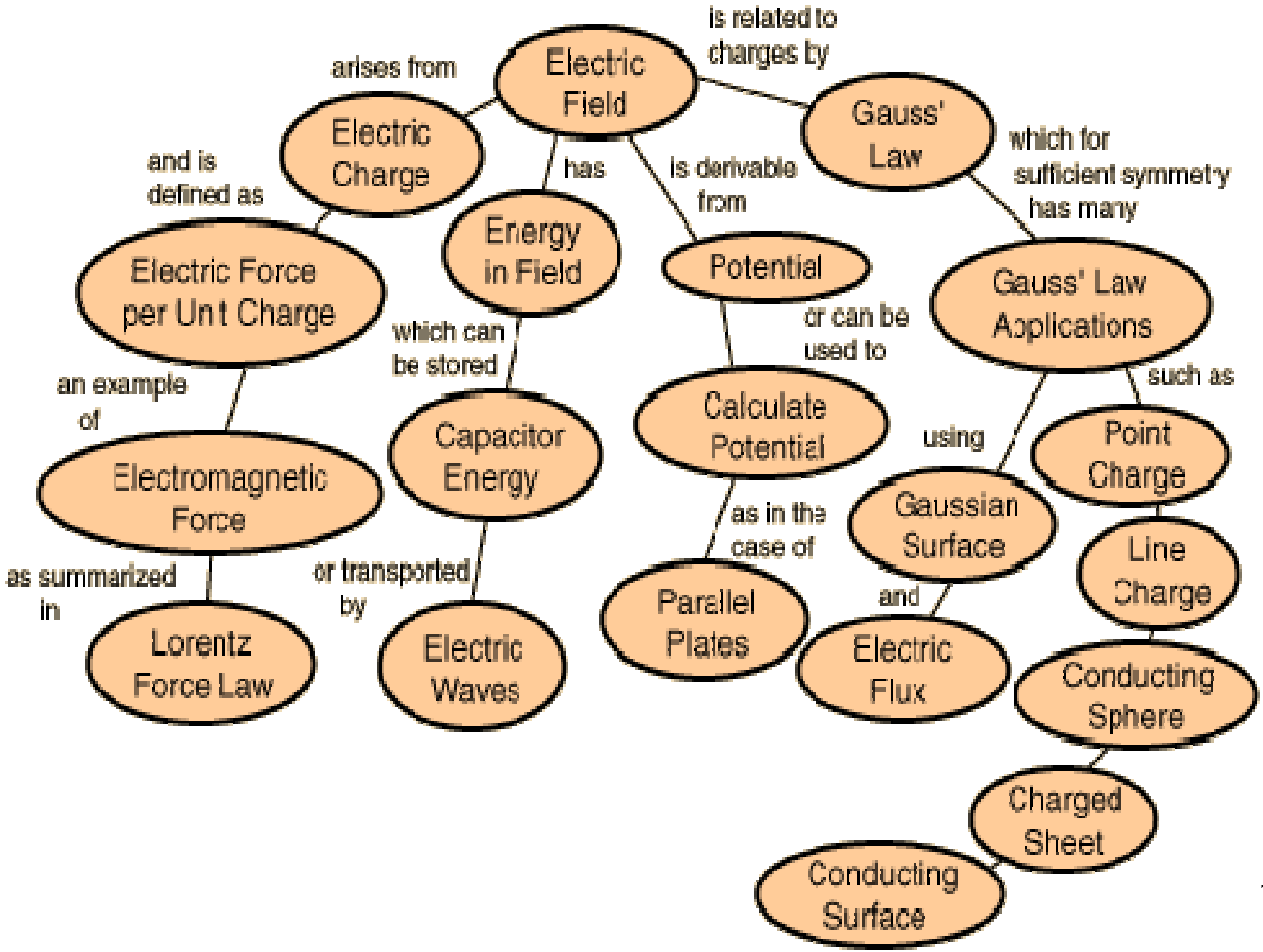

Electric Field
arises from
Electric Charge
and is defined as
Electric Force per Unit Charge
an example of
Electromagnetic Force
as summarized in
Lorentz Force Law
has
Energy in Field
which can be stored
Capacitor Energy
or transported by
Electric Waves
is derivable from
Potential
or can be used to
Calculate Potential
as in the case of
Parallel Plates
is related to charges by
Gauss' Law
which for sufficient symmetry has many
Gauss' Law Applications
using
Gaussian Surface
and
Electric Flux
such as
Point Charge
Line Charge
Conducting Sphere
Charged Sheet
Conducting Surface

Electric field E

Force on a probe charge/ probe charge

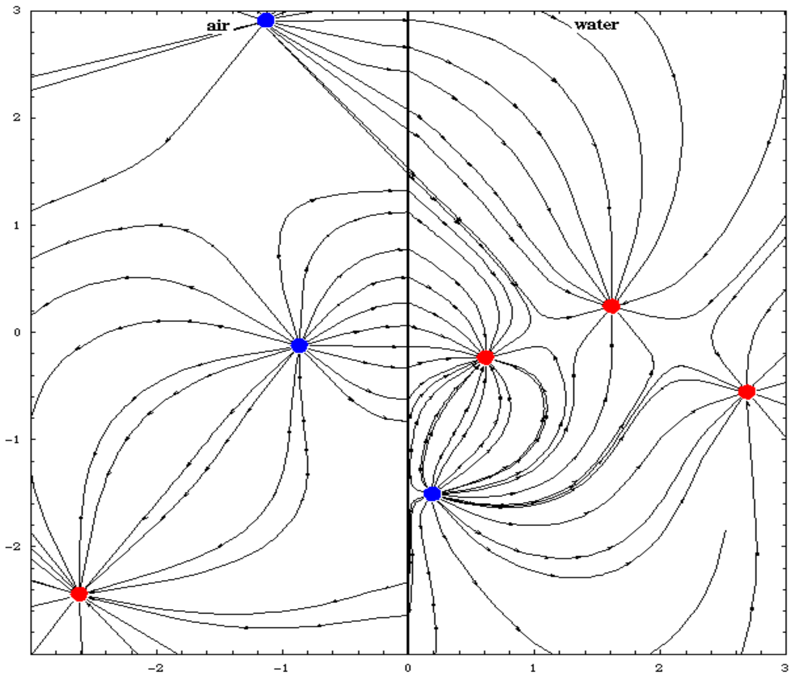
Vector field

Probe charge as small as possible

Dimension = $mlt^{-2}\ q^{-1}$

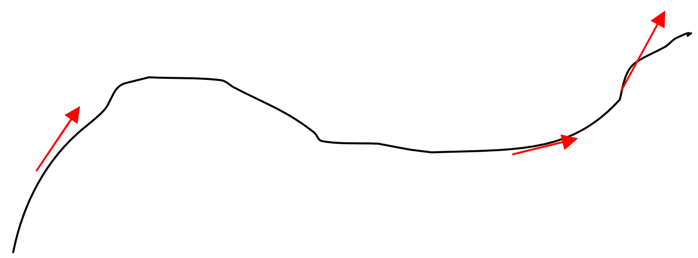
SI Unit = newton/coulomb (N C^{-1})

or volt/meter (V m^{-1}).



Field lines

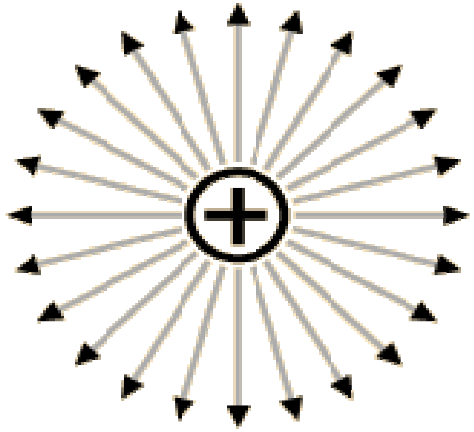
Field line = a line whose tangent at each point is parallel to the vector field at that point



Electric Field **E** is a vector

E(x,y,z) = (*E*x(x,y,z), *E*y(x,y,z), *E*z(x,y,z))

at any point P(x,y,z)

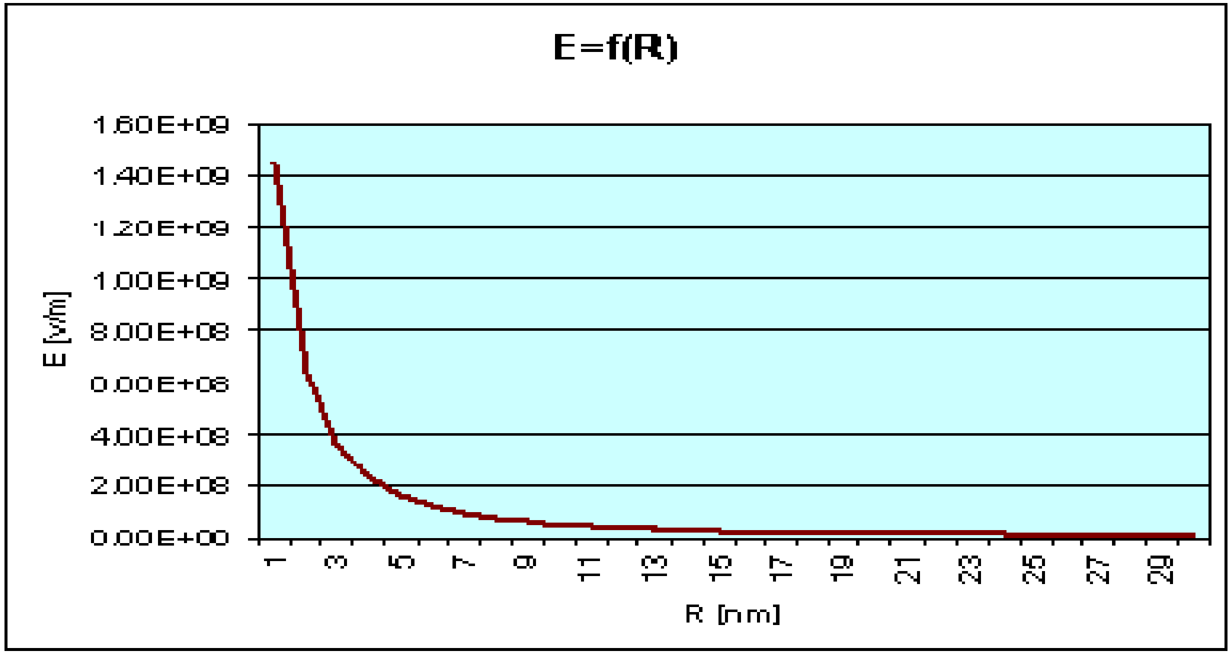


E created by
a point charge Q →

$$E = \frac{1}{\varepsilon_0}\frac{Q}{4\pi r^2}$$

The electric field of a point charge is radially outward from a positive charge

Electric field intensity from
q=1.6X10-19 C (electron)

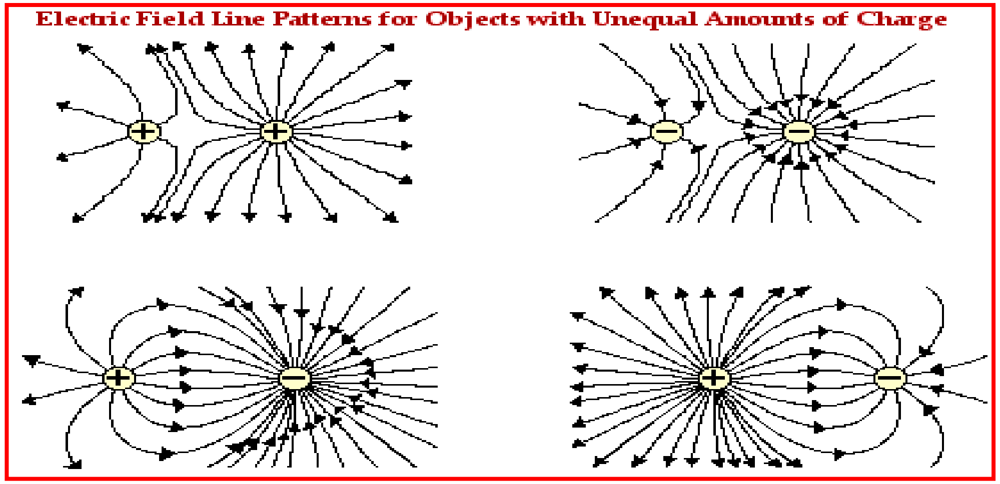


$$E = \frac{1}{\varepsilon_0} \frac{Q}{4\pi r^2}$$

1 point charge

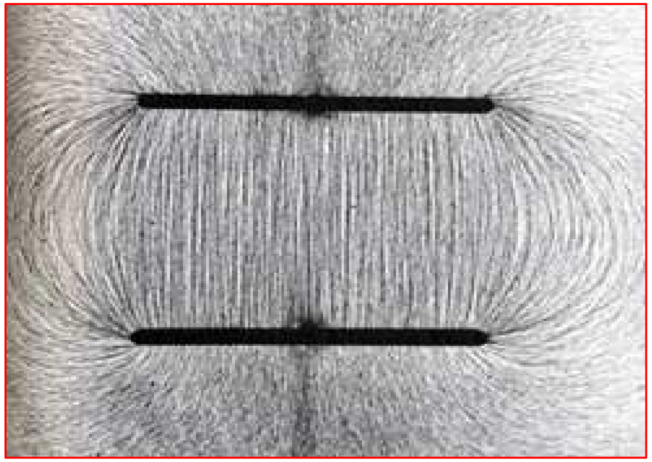
$$E_{total} = \sum_i E_i = E_1 + E_2 + E_3 \ldots$$

N point charges

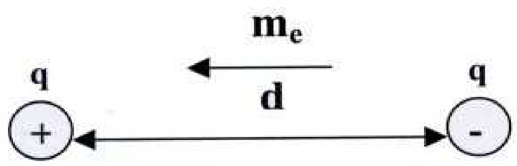


$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{r^2} \hat{\mathbf{r}} \, dV$$

Continuous distribution



Electric dipole moment or **electric dipole**



$$\mathbf{p} = q\,\mathbf{r}$$

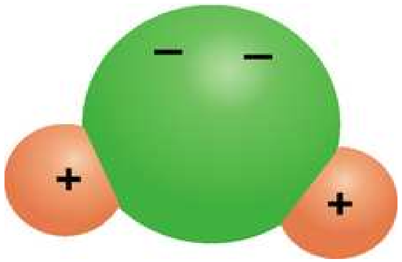
Two point charges

$$\mathbf{p} = \sum_{i=1}^{N} q_i\,\mathbf{r}_i$$

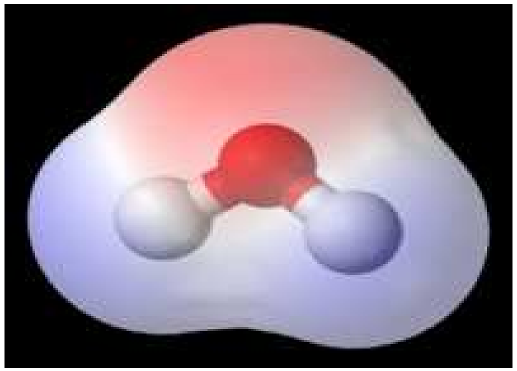
N point charges

$$\mathbf{p} = \int \rho(\mathbf{r}')\,\mathbf{r}'\,d\tau'$$

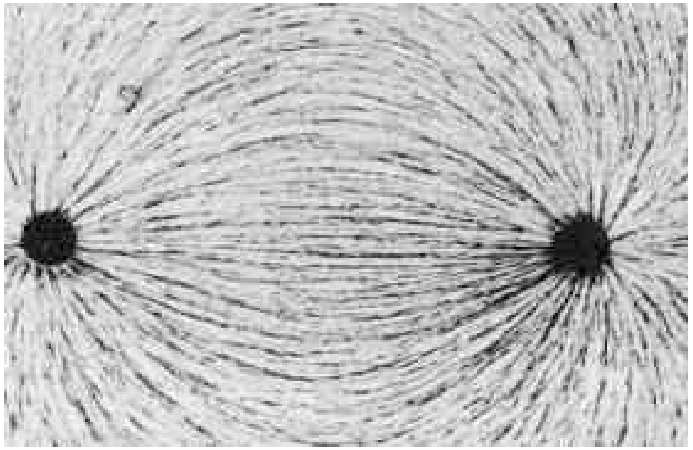
Continuous distribution of charge

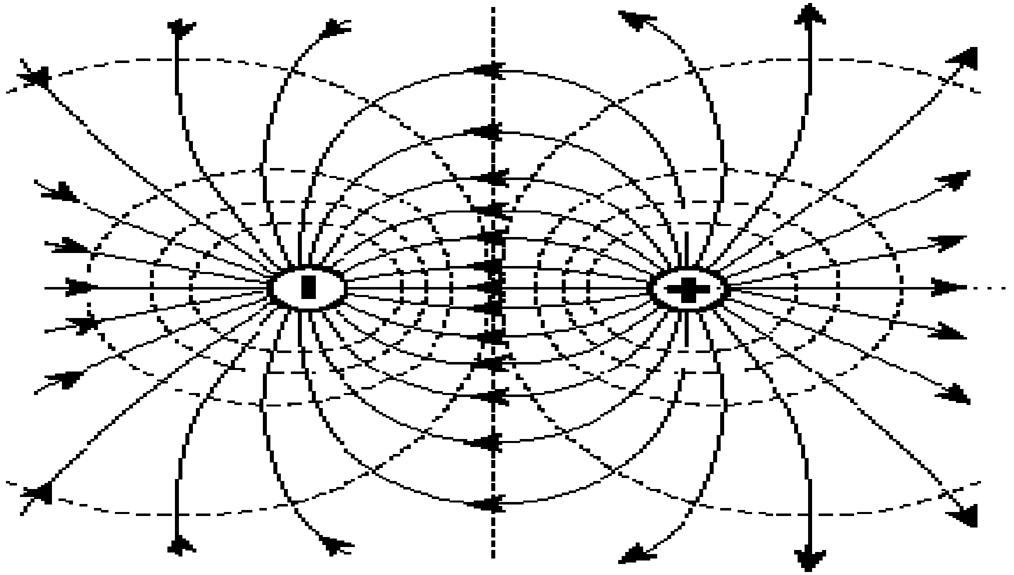


A water molecule is a polar molecule, it has a dipole

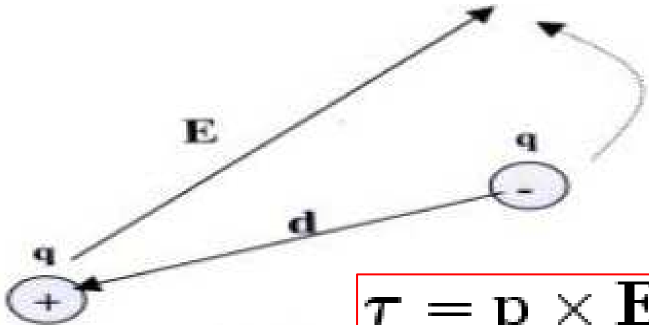


electric dipole field lines





What happens when an electric dipole is in a region of space where there is an electric field E?



$$\boxed{\boldsymbol{\tau} = \mathrm{p} \times \mathbf{E}}$$

$$\boxed{U = -\mathrm{p} \cdot \mathbf{E}}$$

Filed, Potential, Energy

	Particle property	Relationship	Field property
Vector quantity	***Force (on 1 by 2)*** $\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{\mathbf{r}}_{21}$	$\mathrm{F}_{12} = q_1\mathrm{E}_{12}$	***Electric field (at 1 by 2)*** $\mathbf{E}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r^2}\hat{\mathbf{r}}_{21}$
Relationship	$\mathbf{F}_{12} = -\nabla U_{12}$	$U_{12} = q_1 V_{12}$	$\mathbf{E}_{12} = -\nabla V_{12}$
Scalar quantity	***Potential energy (at 1 by 2)*** $U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$		***Potential (at 1 by 2)*** $V_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r}$

Electric Field E (a vector) as Gradient of a Potential V (a scalar)

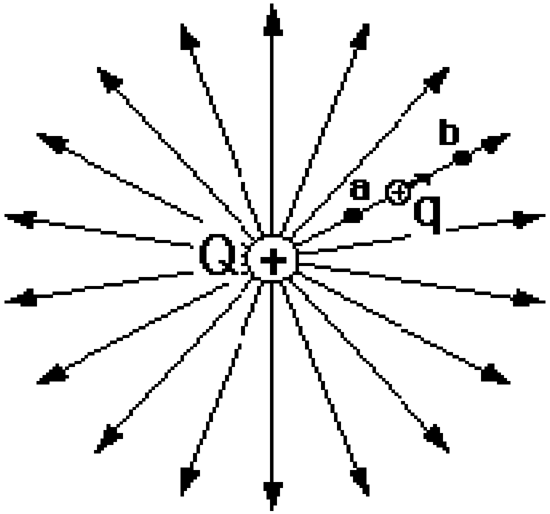
$$E = iE_x + jE_y + kE_z = -i\frac{\partial V}{\partial x} - j\frac{\partial V}{\partial y} - k\frac{\partial V}{\partial z}$$

$$= -\left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right]V$$

$$E = -\nabla V$$ **Gradient or** ∇

Vector field **E** is always perpendicular to a surface where potential V is constant

Point Charge Field E, Potential V, work W



$$\mathbf{E}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r^2}\hat{\mathbf{r}}_{21}$$

$$V_a - V_b = kQ\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

$\boxed{\text{Take limit as } r_b \to \infty}$

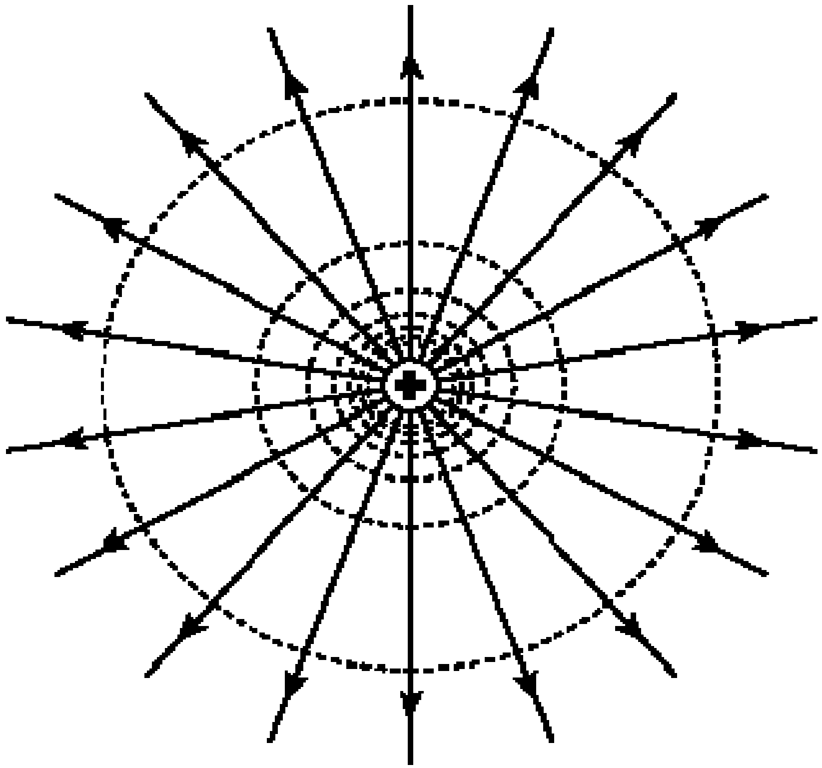
$$E = -\nabla V$$

$$W_{ab} = \int_{r_a}^{r_b} \frac{kQq}{r^2} dr = kQq\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

Potential: the case of a point charge

Potential V of a point charge

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$$



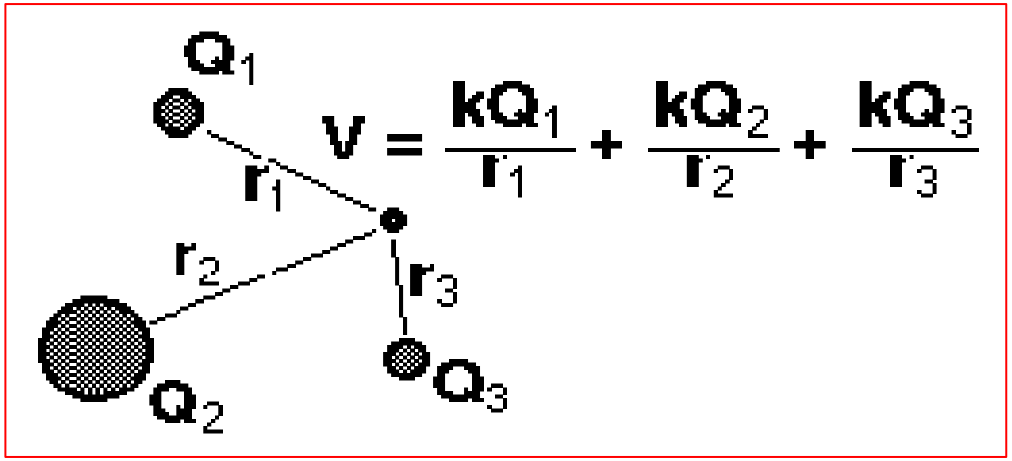
Vector Field E created by Q is radially directed outward

Scalar Potential V is proportional to 1/r

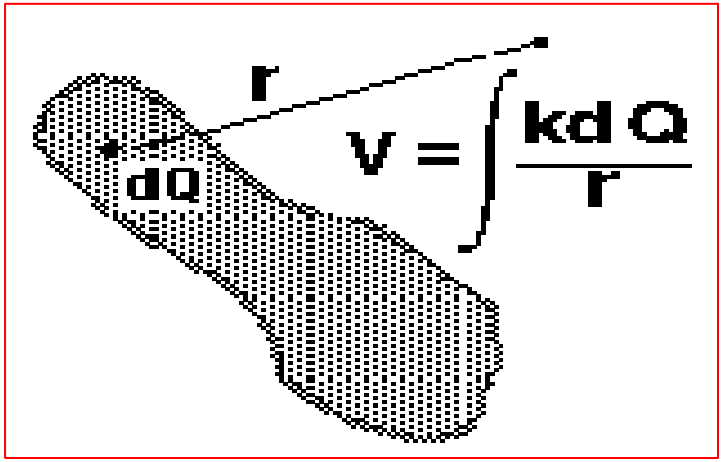
Equipotential surfaces are sphere centred on Q

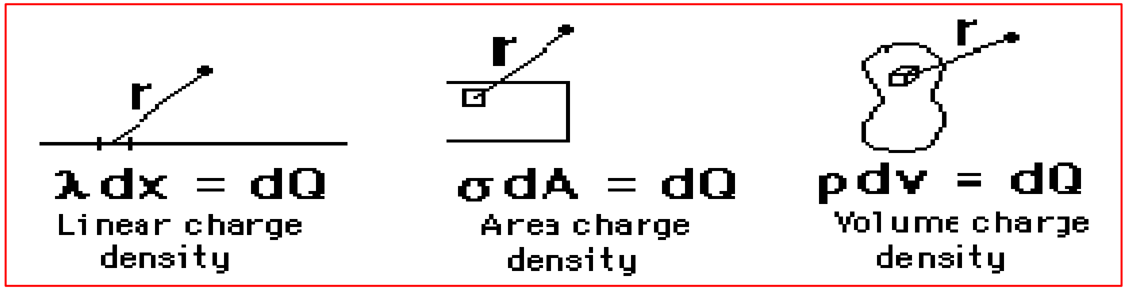
POTENTIAL

3 point charges

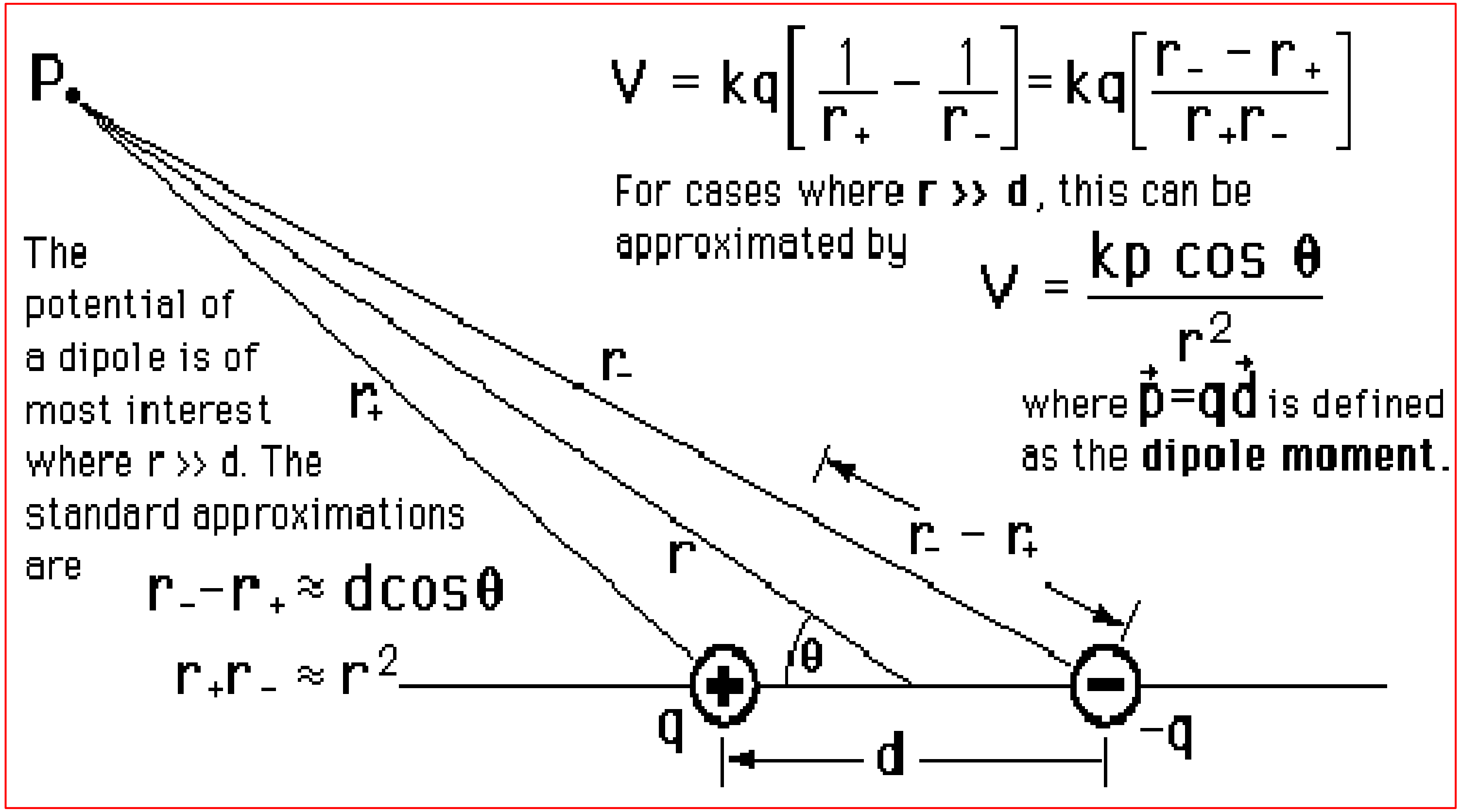


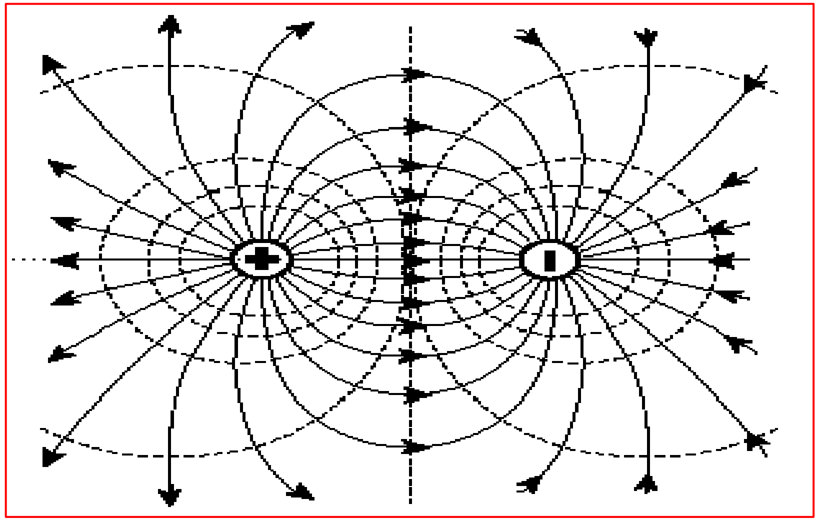
Continuous distribution



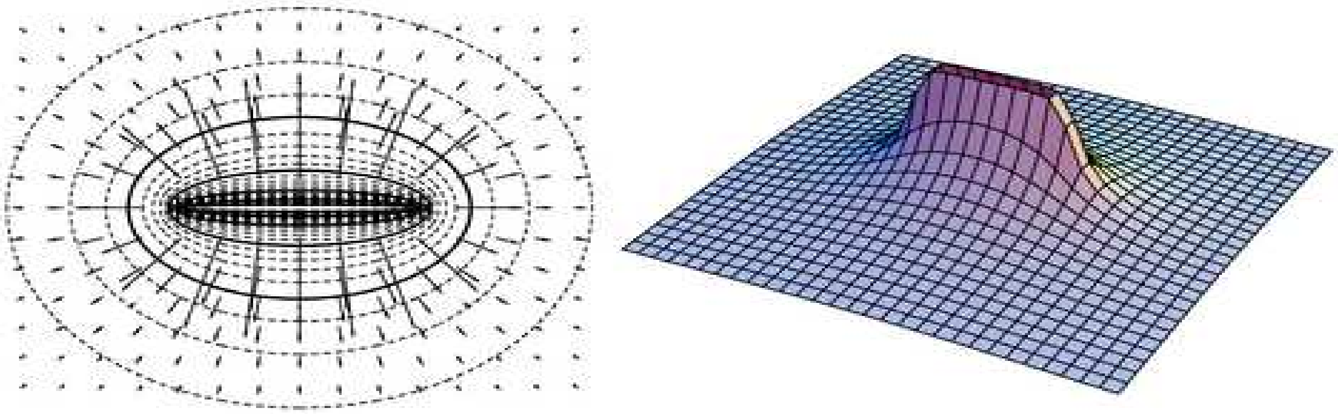


Electric Dipole Potential

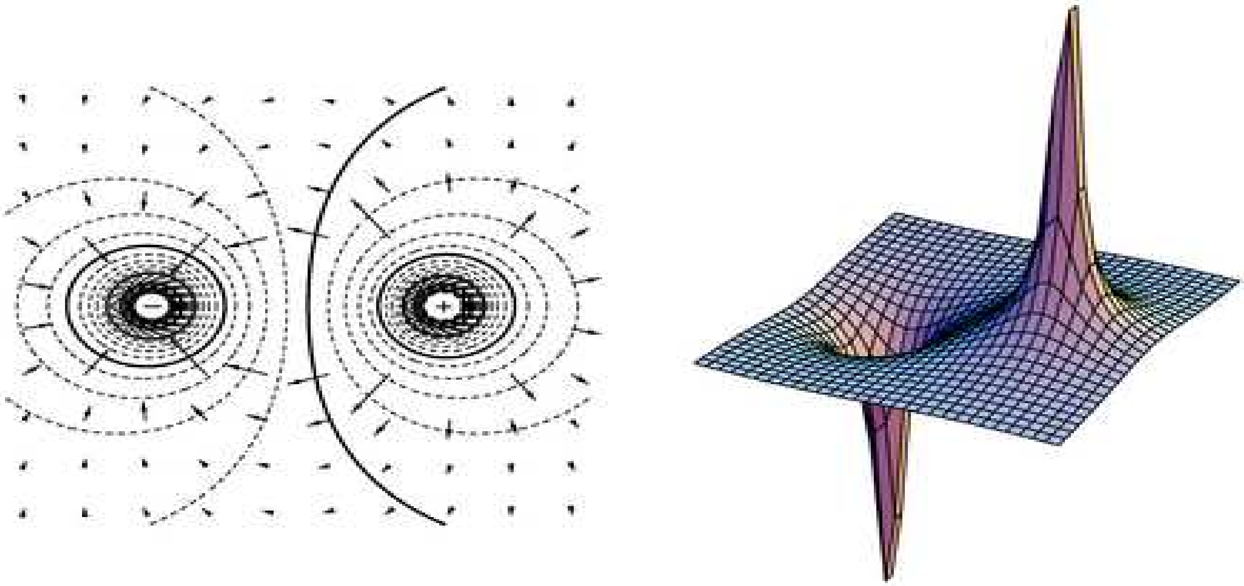




Two-dimensional field and voltage patterns

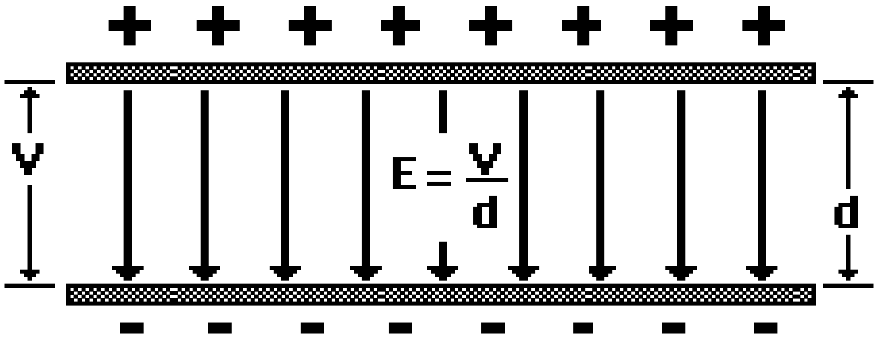


uniformly charged rod



dipole

Constant Electric Field and its Voltage



$$Ed = \frac{Fd}{q} = \frac{W}{q} = \Delta V$$

For constant electric field.

$$E = \frac{F}{q}$$

$$W = q\Delta V$$

General definition relationships

$$E = \frac{V}{d}$$

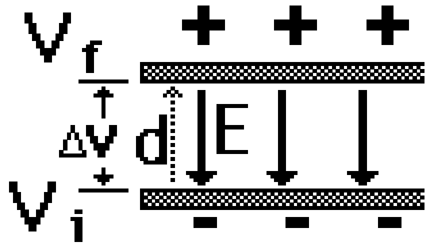
$$V = Ed$$

Constant field special case relationships

Voltage Difference and Electric Field

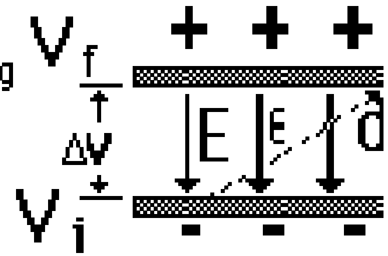
$$V_f - V_i = \frac{Fd}{q} = -Ed$$

Moving a charge from bottom to top plate requires work and raises voltage.



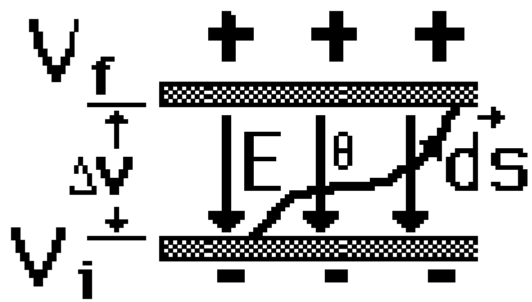
$$V_f - V_i = \frac{\vec{F}\cdot\vec{d}}{q} = -\vec{E}\cdot\vec{d} = -Ed\cos\theta$$

Moving a charge along the slanted line gives the same change in voltage.



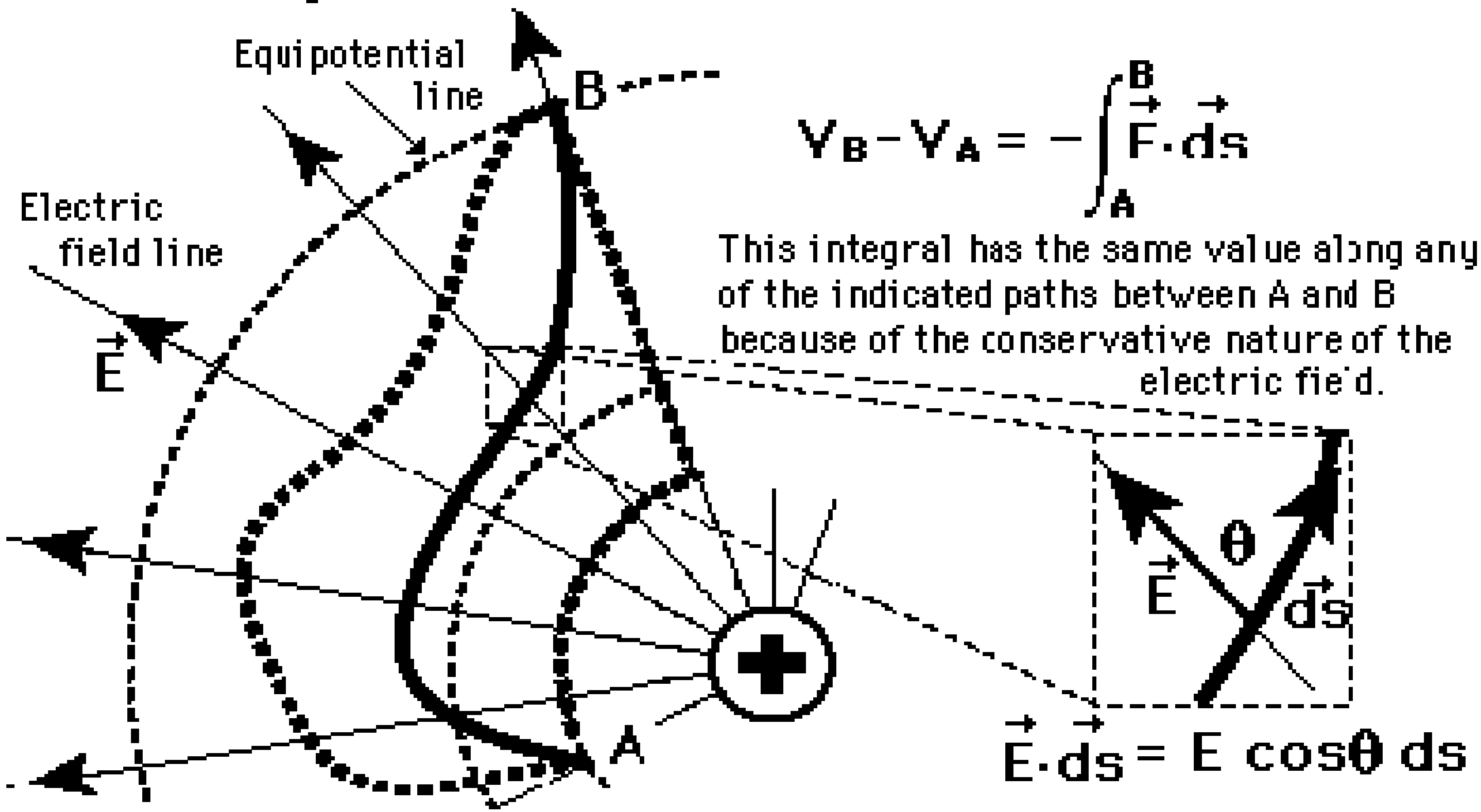
$$V_f - V_i = -\int \vec{E}\cdot d\vec{s}$$

Moving a charge along the curved path indicated would require the integral to calculate, but in this case would give the same voltage difference.

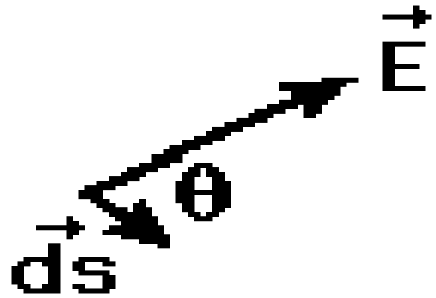


Voltage from Electric Field

$$\frac{dW}{q} = \frac{\vec{F}\cdot\vec{ds}}{q} = \vec{E}\cdot\vec{ds}$$



Electric Field from Voltage



$$dV = -\vec{E}\cdot\vec{ds} = -E_s\, ds$$

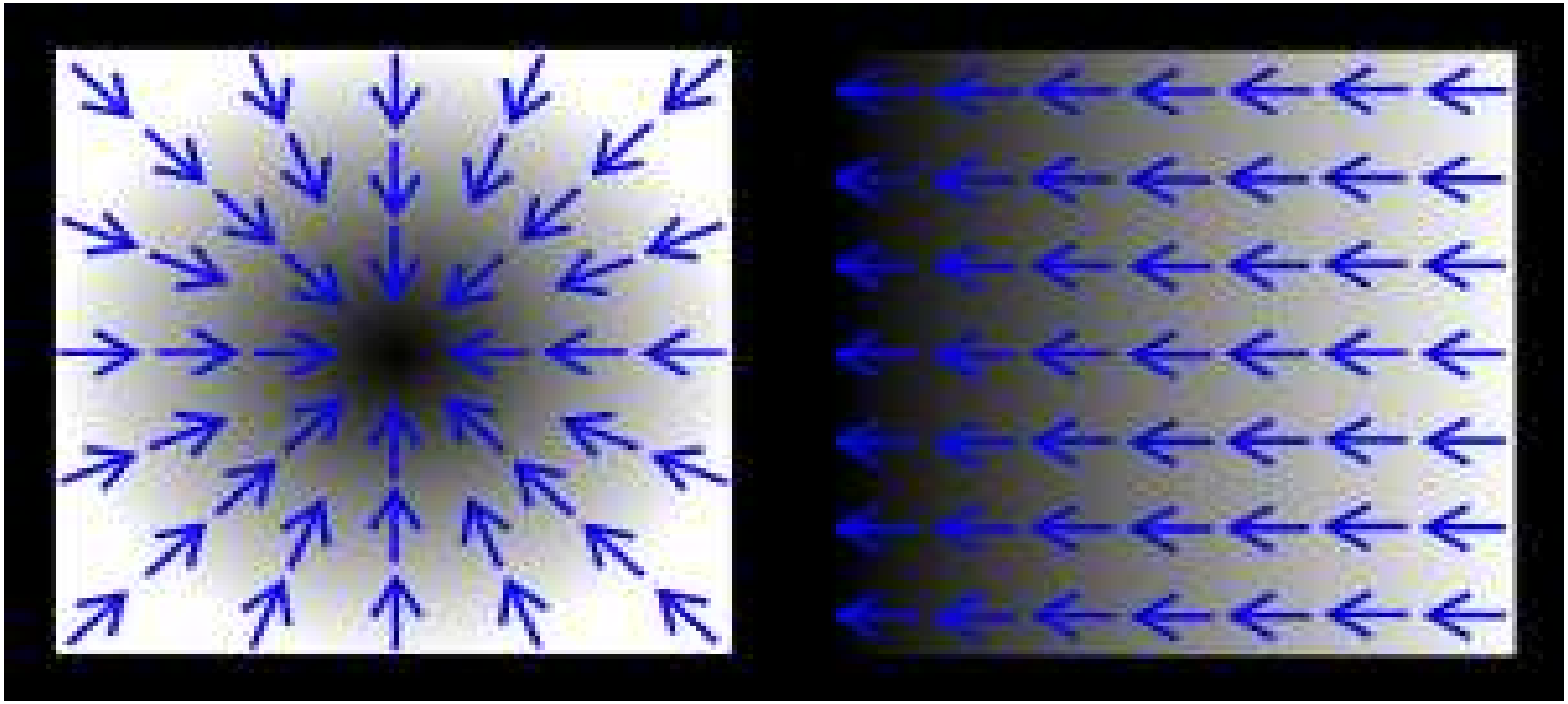
Evaluate voltage change dV along the direction of ds

$$E_s = -\frac{dV}{ds} \text{ along ds, or } \quad E_s = -\frac{\partial V}{\partial s}$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

Gradient

Electric Potential as gradient of electrostatic field



Scalar field (e.g. Temperature) is in black and white,

black representing higher values

Its corresponding gradient is represented by arrows.

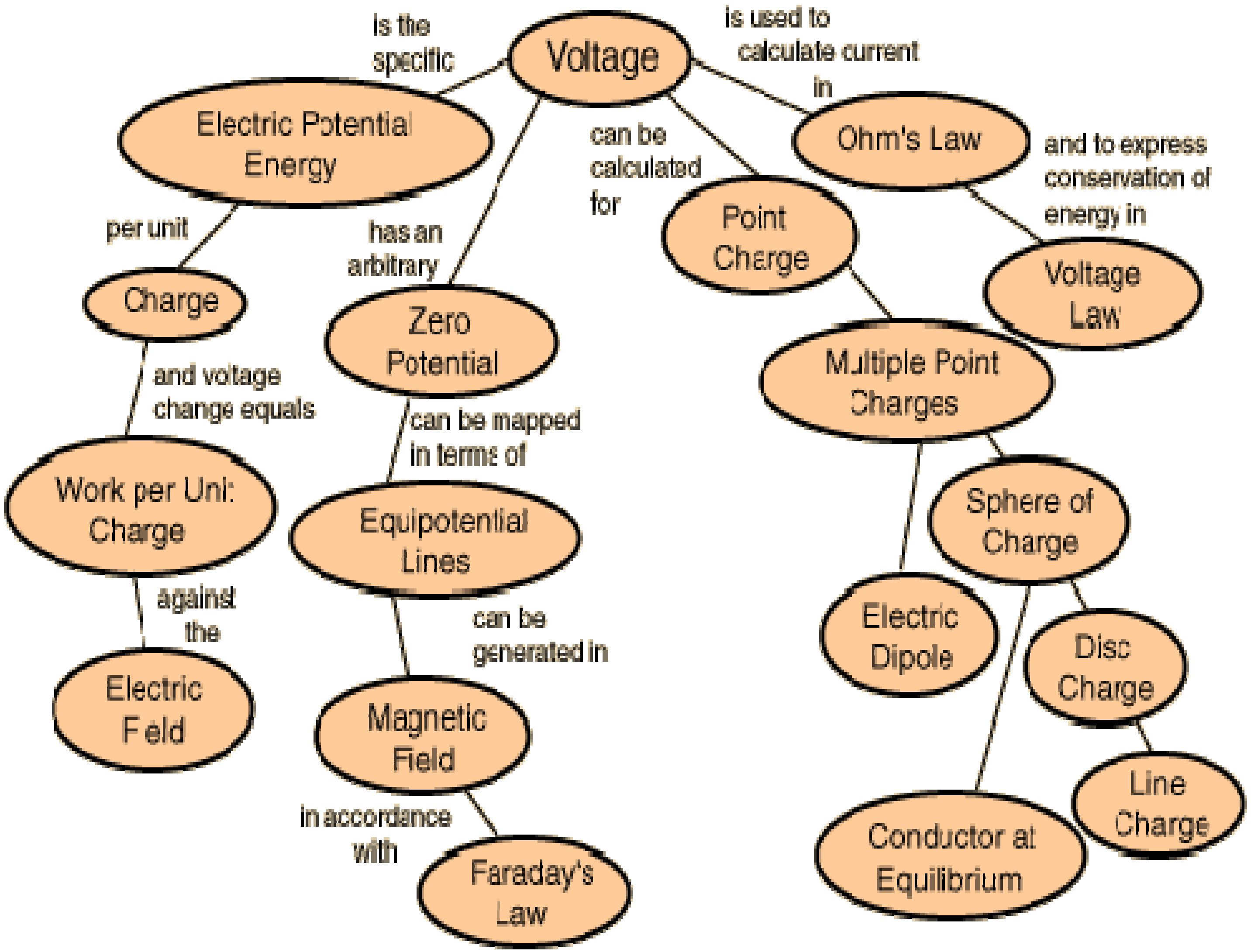
GRADIENT

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_n}\right). \qquad x = (x_1, \dots, x_n) \qquad \vec{\nabla} f$$

$$(\nabla f)_x \cdot v$$

In Cartesian coordinates

$$\nabla f(x, y, z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$


Voltage
is the specific
Electric Potential Energy
per unit
Charge
and voltage change equals
Work per Unit Charge
against the
Electric Field
has an arbitrary
Zero Potential
can be mapped in terms of
Equipotential Lines
can be generated in
Magnetic Field
in accordance with
Faraday's Law
can be calculated for
Point Charge
Multiple Point Charges
Electric Dipole
Sphere of Charge
Conductor at Equilibrium
Disc Charge
Line Charge
is used to calculate current in
Ohm's Law
and to express conservation of energy in
Voltage Law