Introductory General Physics (Sassi-Smaldone)

Gulu University

Naples FEDERICO II University

1

Measurements & Numbers

NUMBERS

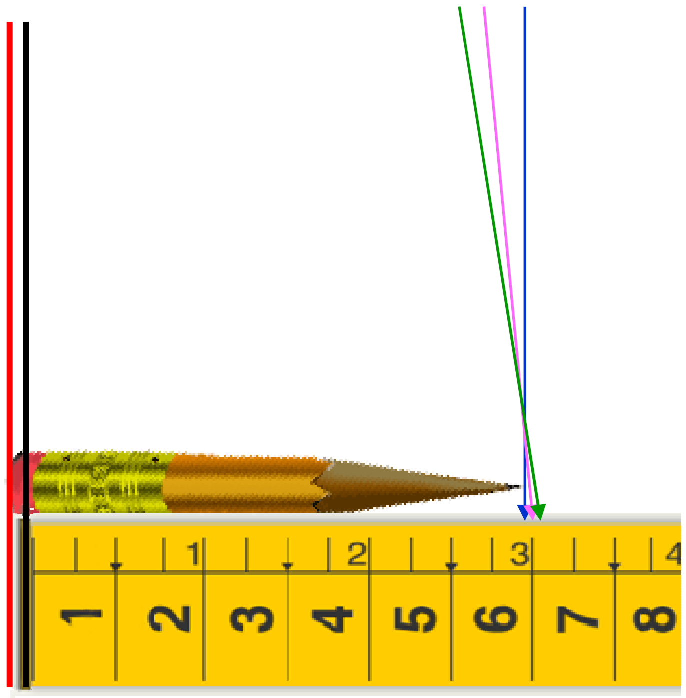
Are 12.32 meters equal to 12.32000 meters ?

Yes for a M**athematician** **NO** for a **physicist**, a **chemist**, a **biologist** etc. (an **experimentalist**) !

Unit of measure

It's the result (**direct** or **indirect**) of a **measure** the (*significant*) **digits** do have a precise meaning

Direct Measure of a Quantity



- Comparison with a **UNIT**
  - 5.9 cm
  - 6.0 cm
  - 6.1 cm
  - **....** cm
  - **....** cm

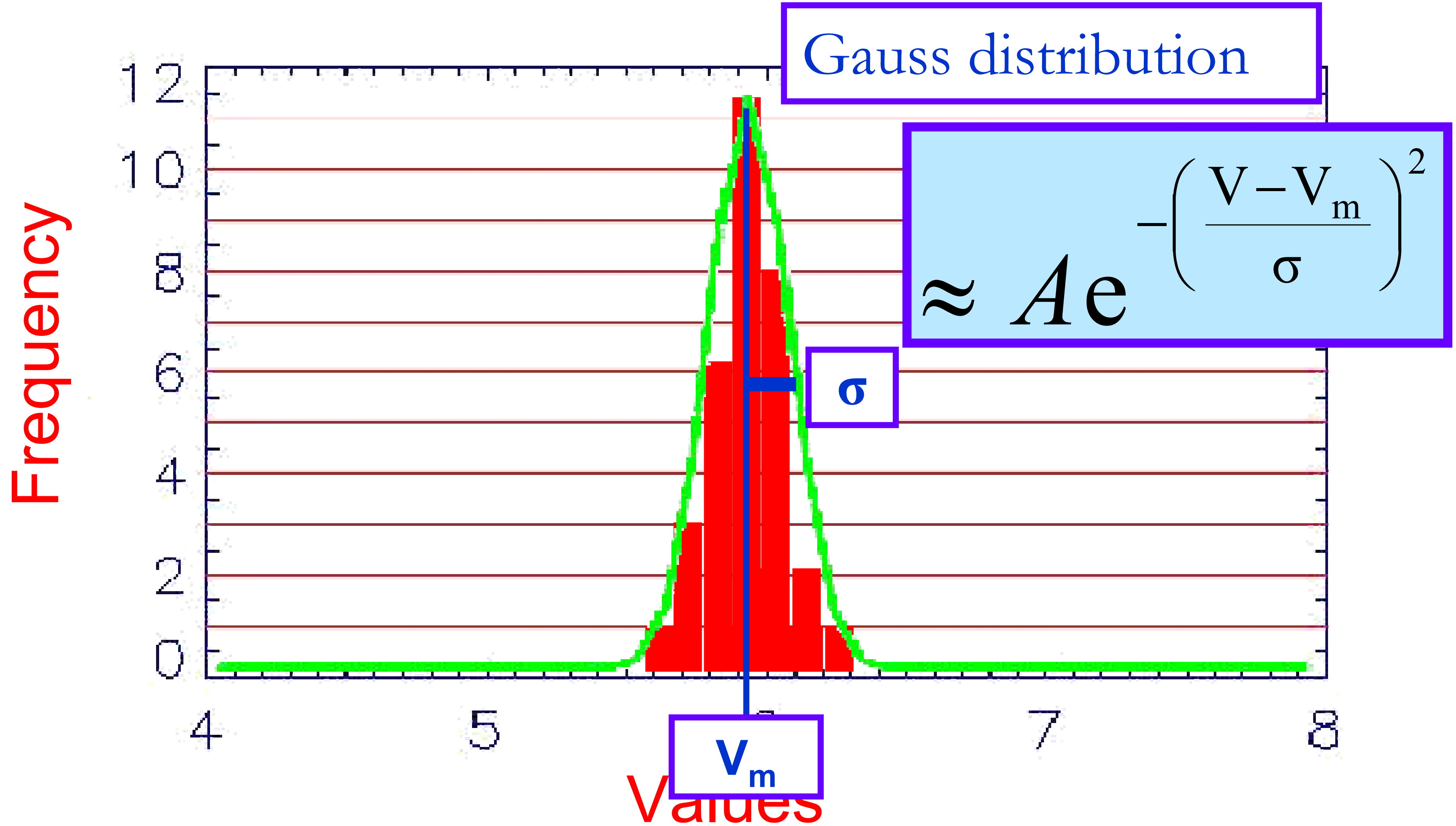
Random Errors (±)

Systematic Errors

If found, can be corrected

(offset, calibration, procedure, measure conditions, preparation, ..)

Distribution of Measures (histogram)

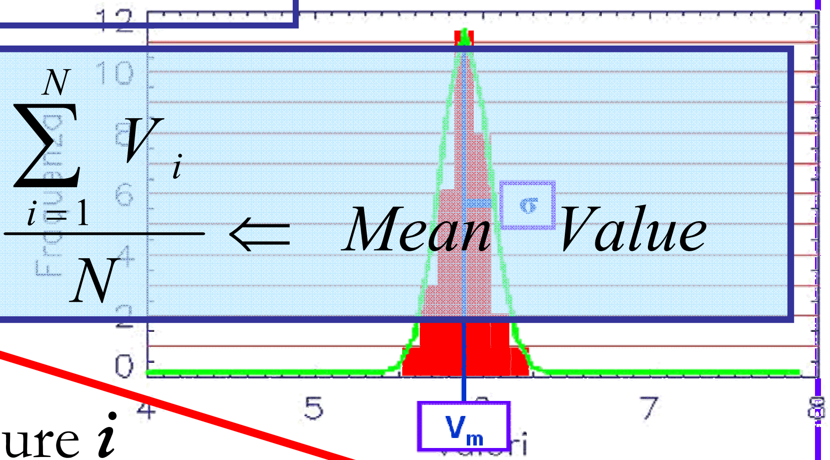


What is the length of the pencil ?

V_1=5.9 V_2=6.1 V_3=6.0 V_4=5.9 V_5=5.8 V_6=6.2 V_7=5.6 …. V_i=…. ….

N measurements

$$\boxed{V_m = \frac{V_1 + V_2 + V_3 + \ldots + V_i + \ldots}{N}} = \frac{\sum_{i=1}^{N} V_i}{N} \Leftarrow \quad Mean \quad Value$$

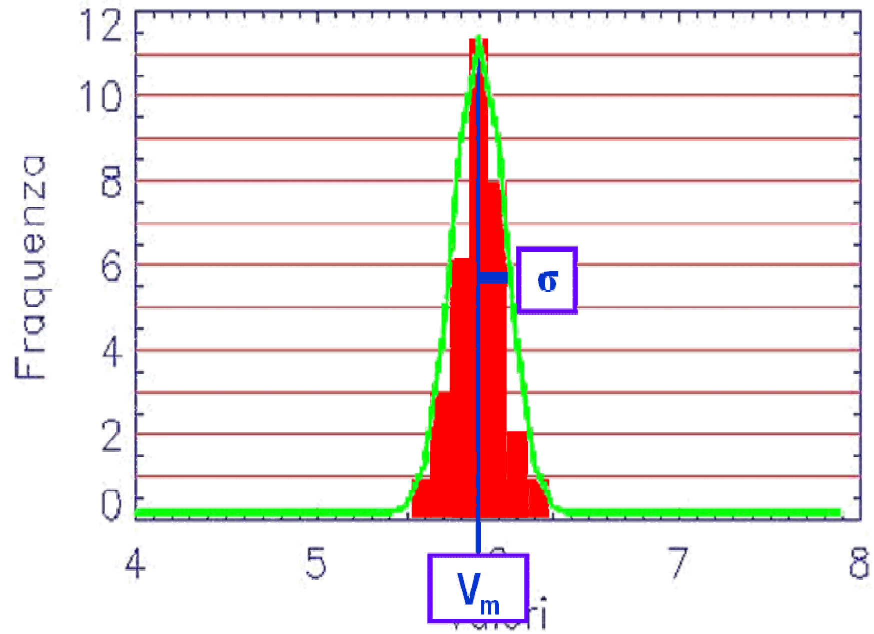


$(V_i - V_m)$ **deviation** (from average) of measure $\boldsymbol{i}$

$$\sum_{i=1}^{N}(V_i - V_m) = (V_1 - V_m) + (V_2 - V_m) \ldots + (V_i - V_m) + \ldots = V_1 + \ldots + V_i + \ldots - N\boxed{V_m} = 0$$

What is the length of the pencil ?

An indicator of the histogram width is needed to synthesise how **good** the measure is



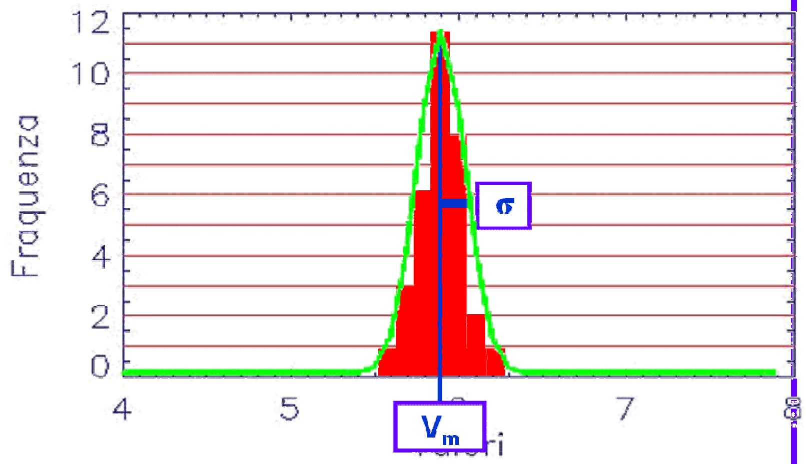
$$\sigma = \sqrt{\frac{(V_1 - V_m)^2 + (V_2 - V_m)^2 + \ldots + (V_i - V_m)^2 + \ldots}{N(N-1)}} = \sqrt{\frac{\sum_{i=1}^{N} (V_i - V_m)^2}{N(N-1)}}$$

Standard Deviation σ (or **error**)

What is the length of the pencil ?

The **measure** is written as

$$\boxed{V_m \pm \sigma}$$



It means that, when a new measure is taken under the same conditions, the new value V has a probability as:

- 68% $(V_m - \sigma) \leq V < (V_m + \sigma)$
- 95% $(V_m - 2\sigma) \leq V < (V_m + 2\sigma)$

(Numerical Approximation)

Rules to round a number

If the first digit to cancel (**control digit**) is:

a) <5 → the digits to keep do not change (**approx. by defect**)

b) >5 → the last digit to keep is increased by 1 (**approx. by excess**)

c) =5 → the last digit to keep is rounded to the even digit

d) =50 → rounding by excess or defect

Ex:

17.6712 at 3 digits is (**b**) **17.7**; **17.6472** at 3 digits is (**a**) **17.6**

17.6572 at 3 digits is (**c**) **17.6**; **17.7572** at 3 digits is (**c**) **17.8**

17.7502 at 3 digits is (**d**) **17.7** or **17.8**

Writing of the Measure's Value

Error (uncertainty) explicit: $x \pm \Delta x$ $(x \pm \sigma)$

Error (uncertainty) implicit, given by the last significant digit: 32.5**4** kg $\rightarrow$ **±0.005** kg 32.**5** kg $\rightarrow$ **±0.05** kg; 3**2** kg $\rightarrow$ **±0.5** kg

- Numbers used in calculations may have an extra significant digit, with respect to what are requested for the final value, to minimize inaccuracies coming from rounding
- Measure and error must be written with the same unit
- When calculating, the result has to be rounded according to the number of significant digits of the data that have the minimum of significant digits

$\Delta x = \sigma$	**Absolute Error**
$\Delta x / x$	**Relative Error** or **Fractional Uncertainty**
$100 \times \Delta x / x$	**Percentage Error**

Examples of Measures

	Length of street x=4 km $\Delta x=\sigma=2$ m	Length of rafter x=1 m $\Delta x=\sigma=1$ mm
to write	4000 ± 2 m	1.000 ± 0.001 m
ε_r	2/4000=0.0005	0.001/1=0.001
$\varepsilon_{\%}$	0.05%	0.1%

Which is the more accurate (precise) measure ?

Scientific Notation

$10^0=1$ – $10^1=10$ – $10^2=100$ – $10^3=1000$ – $10^4=10000$ $10^{-1}=0.1$ – $10^{-2}=0.01$ – $10^{-3}=0.001$ – $10^{-4}=0.0001$

Properties: $10^n \times 10^m = 10^{n+m}$ – $10^n : 10^m = 10^{n-m}$

$10^2 \times 10^4 = 10^{2+4} = 10^6$ – $10^2 \times 10^{-5} = 10^{2+(-5)} = 10^{-3}$

$10^2 : 10^4 = 10^{2-4} = 10^{-2}$ – $10^2 : 10^{-5} = 10^{2-(-5)} = 10^7$

N. S.: y.xxx $\times 10^m$ with $1 \le y \le 9$ signif. digits

- Simplification of calculations (much less mistakes with pocket calculators).
- Better control of significant digits !!!

Examples of scientific notation

33.5 kg in grams $=$**33500 g**

3.35×10^{1} kg in grams $=$**3.35×10^{4} g**

$$\frac{7.52\times10^{3}\bullet3.242\times10^{-7}\bullet1.7\times10^{2}}{2.34\times10^{5}\bullet3.14\times10^{-4}}=$$

Power of 10: **3-7+2-(5-4) = -3**

$$\frac{7.52\bullet3.242\bullet1.7}{2.34\bullet3.14}=5.640716424$$

$=$**5.6×10^{-3}**

Order of Magnitude of a Measure

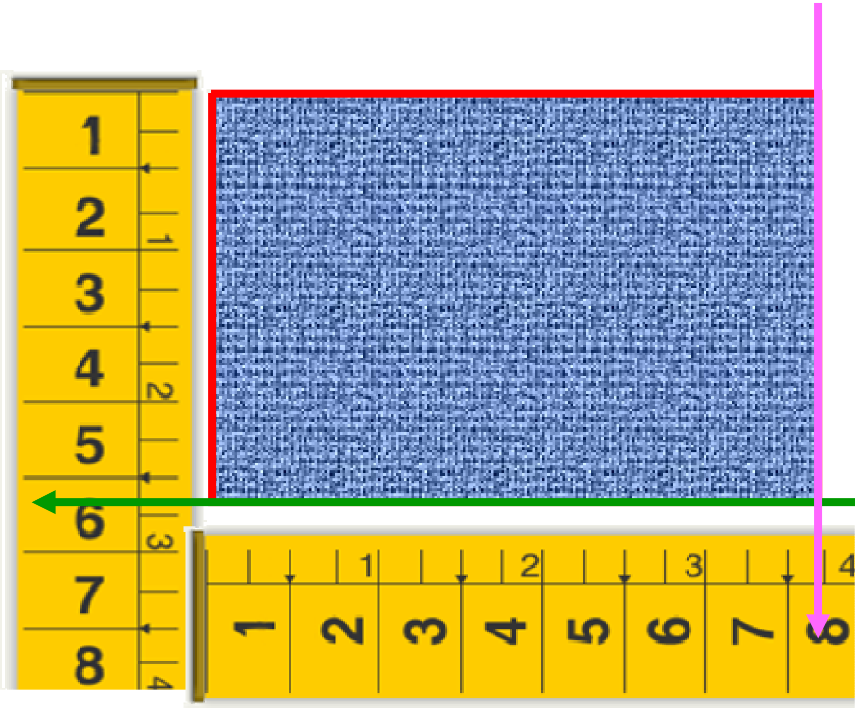
Order of Magnitude of **X**

$\rightarrow 10^{(\log x)}$ **approximated to unity**

O o M of 23 = 10 ($\log_{10} 23 = 1.36$)

O o M of 850 = 10^3 ($\log_{10} 850 = 2.92$)

Indirect Measure of a Physics Quantity



Surface S = **B**ase × **H**igh

B=7.4±0.15 cm

H=5.3±0.15 cm

S_{min}=7.25×5.15=**37.3375** cm^2

S_{max}=7.55×5.45=**41.1475** cm^2

68% Probability to have **37.3375 ≤ S < 41.1475**

S = 39 ± 2 cm^2 (S ± ΔS)

Error in an Indirect Measure of a Physics Quantity

(propagation of error)

$$y = \mathbf{y} \pm \Delta_y \quad ; \quad x = \mathbf{x} \pm \Delta_x \quad ; \quad z = \mathbf{z} \pm \Delta_z$$

$G = f(x, y, z)$ where f is a relation (law) from physics, math, geometry.

$$\boxed{G = \mathbf{G} \pm \Delta_G}$$

$$\boxed{\mathbf{G} = f(\mathbf{x}, \mathbf{y}, \mathbf{z})}$$

$$\boxed{\Delta_G = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \Delta_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \Delta_y^2 + \left(\frac{\partial f}{\partial z}\right)^2 \Delta_z^2}}$$

Error in an Indirect Measure of a Physics Quantity

(more frequent cases)

$G = a\,x + b\,y$

$$\Delta_G = \sqrt{a^2 \Delta_x^2 + b^2 \Delta_y^2}$$

$G = 3x + 2y$

$x = 4.1 \pm 0.2$

$y = 2.2 \pm 0.4$

$$\Delta_G = \sqrt{9 \bullet 0.04 + 4 \bullet 0.16} = 1$$

$G = 16.7$ **$G = 17 \pm 1$**

$G = x \bullet y$

$$\Delta_G = \sqrt{y^2 \Delta_x^2 + x^2 \Delta_y^2}$$

$G = 9 \pm 1.7$

$G = x/y$

$$\Delta_G = \frac{1}{y^2}\sqrt{y^2 \Delta_x^2 + x^2 \Delta_y^2}$$

$G = 1.9 \pm 0.4$