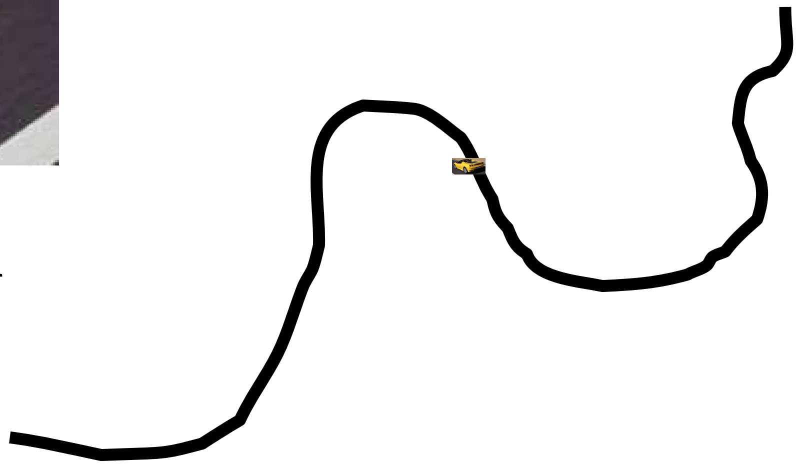
Point-like Object (P. O.)

Is it a Point-like object ?



Is it a Point-like object when

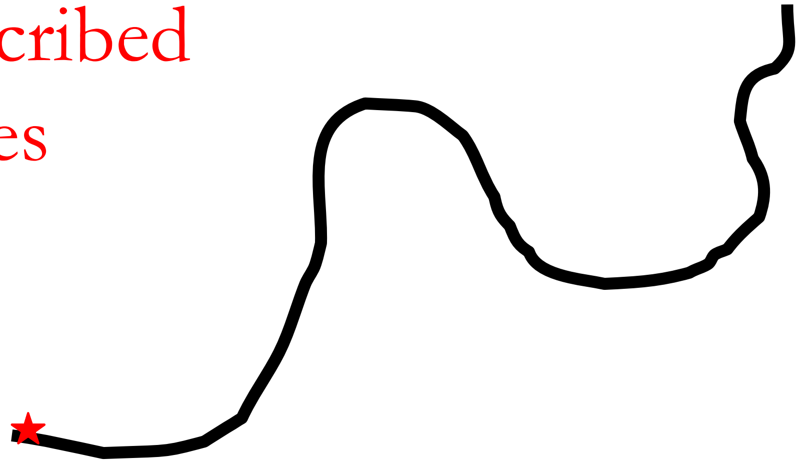
it's small with respect to environment

in which it moves

Position defined by its coordinates at that instant of time

Trajectory (of a P. O.)

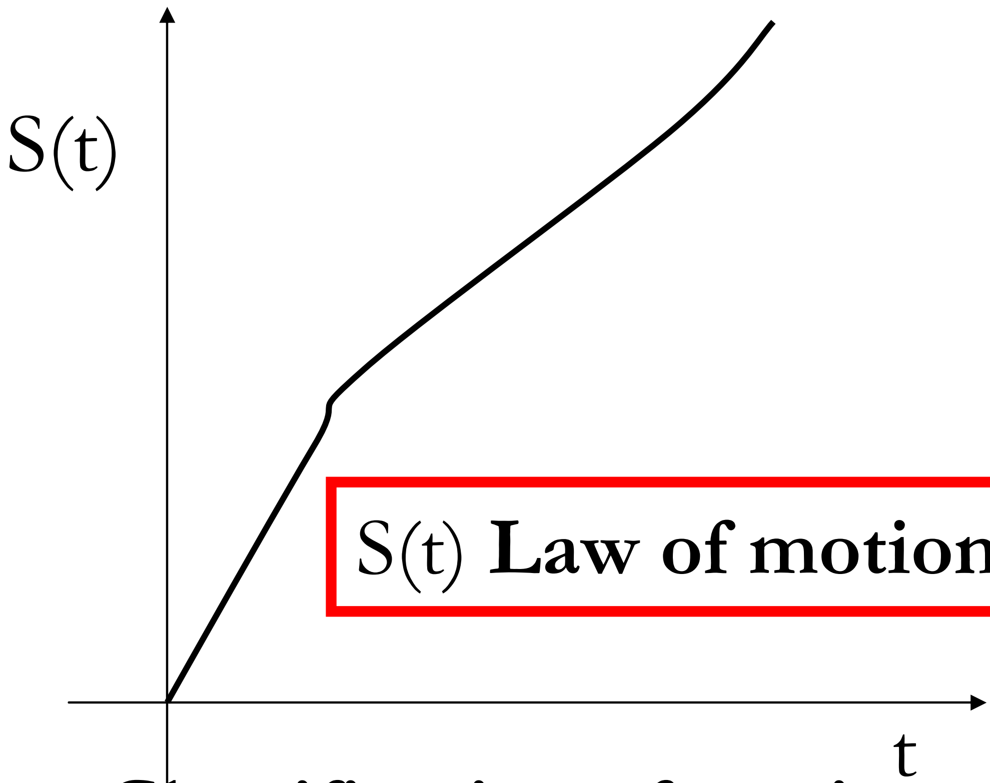
Trajectory: geometric curve described by P.O. in time, when P.O. moves



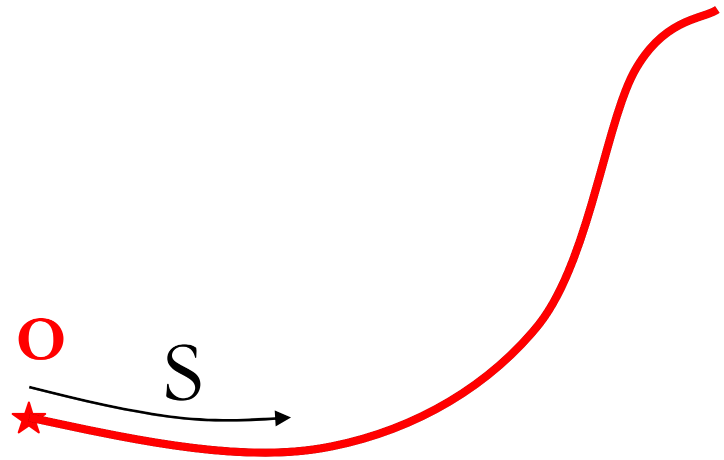
Classification of motion (according to type of traiectory):

straight, circolar, elicoidal, spiral,….

Description of motion



S(t) **Law of motion**



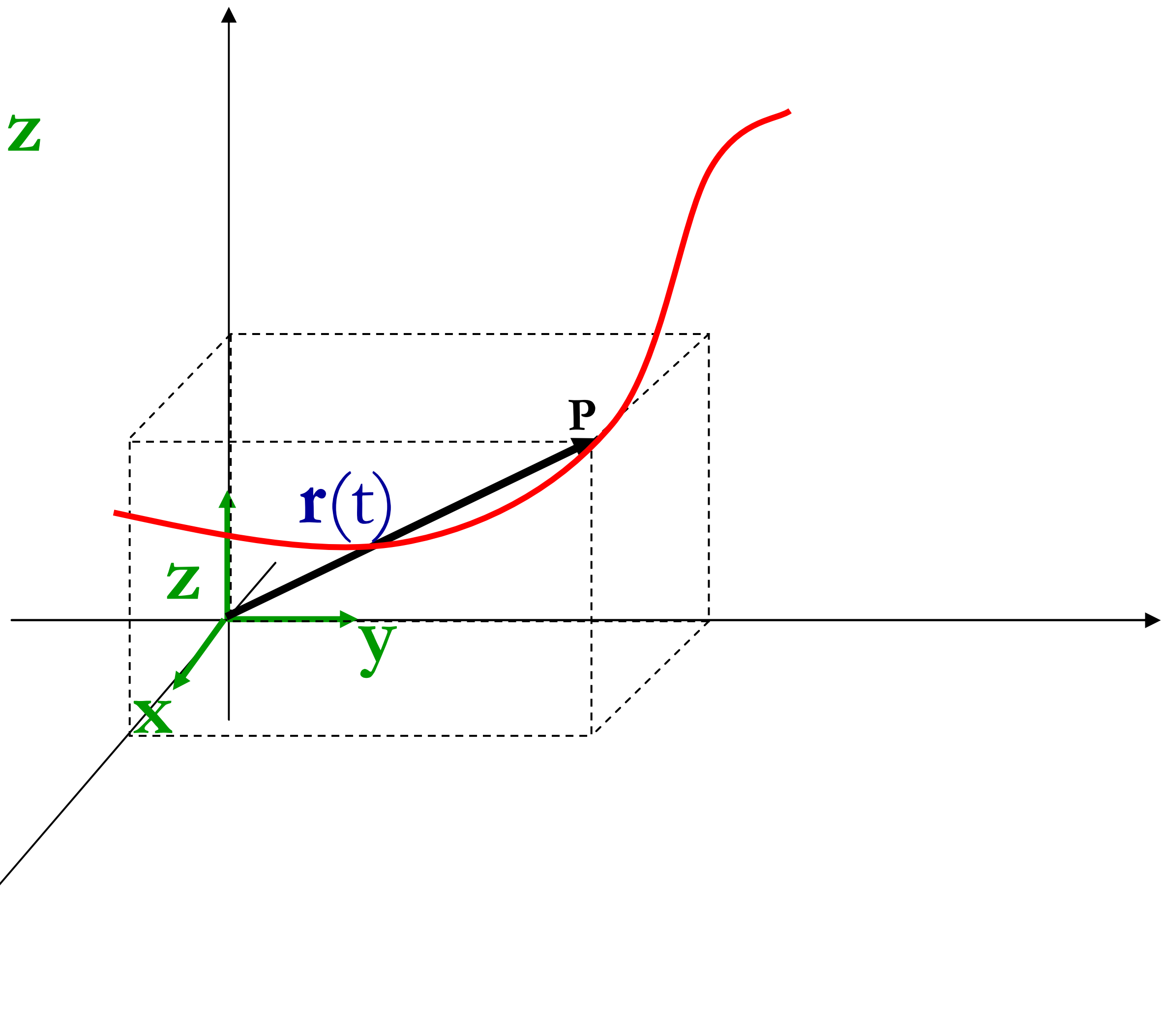
Classification of motion (according S(t))**:**

Uniform, Various, accelerated, retarded, …..

Description of motion

$\mathbf{r}(t) = x(t)\mathbf{x} + y(t)\mathbf{y} + z(t)\mathbf{z}$

1. $x(t)$
2. $y(t)$
3. $z(t)$



Average Velocity

P position at time t, $\mathbf{r}(t)$

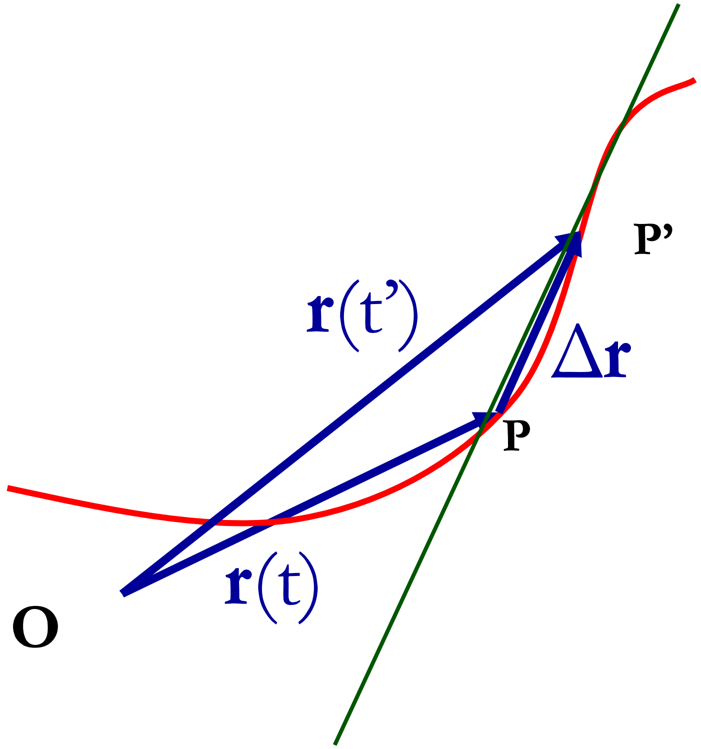
P' position at time t' = t+Δt, $\mathbf{r}(t')$

displacement $\Delta\mathbf{r} = \mathbf{r}(t+\Delta t) - \mathbf{r}(t)$

$$\mathbf{v}_m = \Delta\mathbf{r} / \Delta t$$

$\mathbf{v}_m$ average velocity

direction of $\mathbf{v}_m$ = cord PP'

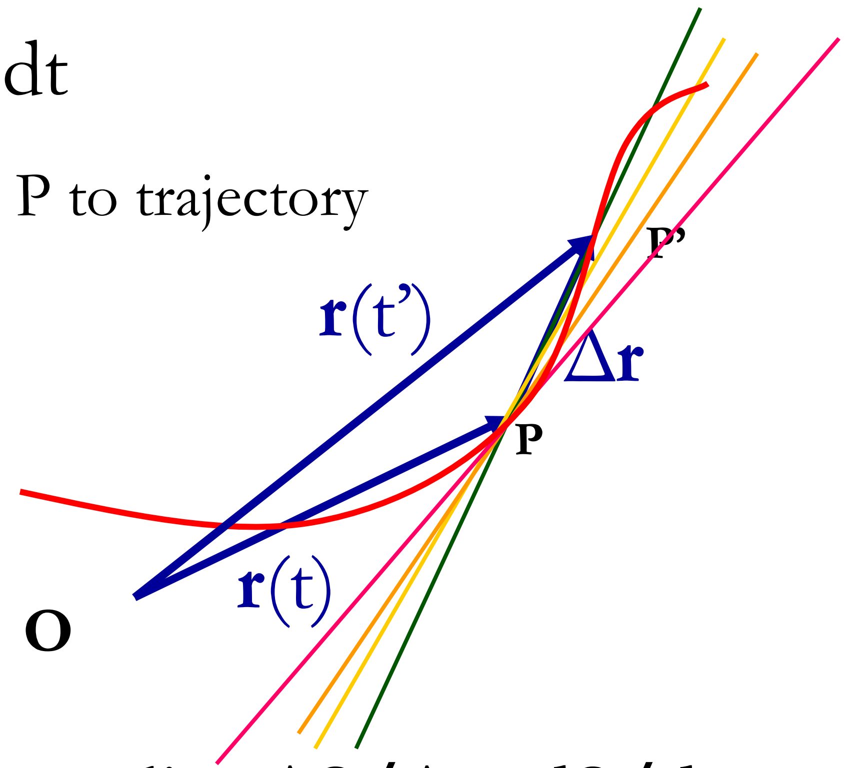


$[\mathbf{v}_m] = l^1 \cdot t^{-1}$; m/s

Instantaneous Velocity

$$\mathbf{v}(t)=\lim_{\Delta t \to 0} \Delta\mathbf{r}/\Delta t = d\mathbf{r}/dt$$

direction of $\mathbf{v}$ = tangent in P to trajectory



when $|\Delta\mathbf{r}|$ gets small

$|\Delta\mathbf{r}| \approx |\Delta S|$ (arch and cord almost the same)

$$v(t)=|\mathbf{v}(t)|= \lim_{\Delta t \to 0} |\Delta\mathbf{r}|/\Delta t = \lim_{\Delta t \to 0} \Delta S/\Delta t=dS/dt$$

if $\boldsymbol{\tau}$ **is unit vector tangent** $\mathbf{v}(t)= v(t)\ \boldsymbol{\tau} =(dS/dt)\ \boldsymbol{\tau}$

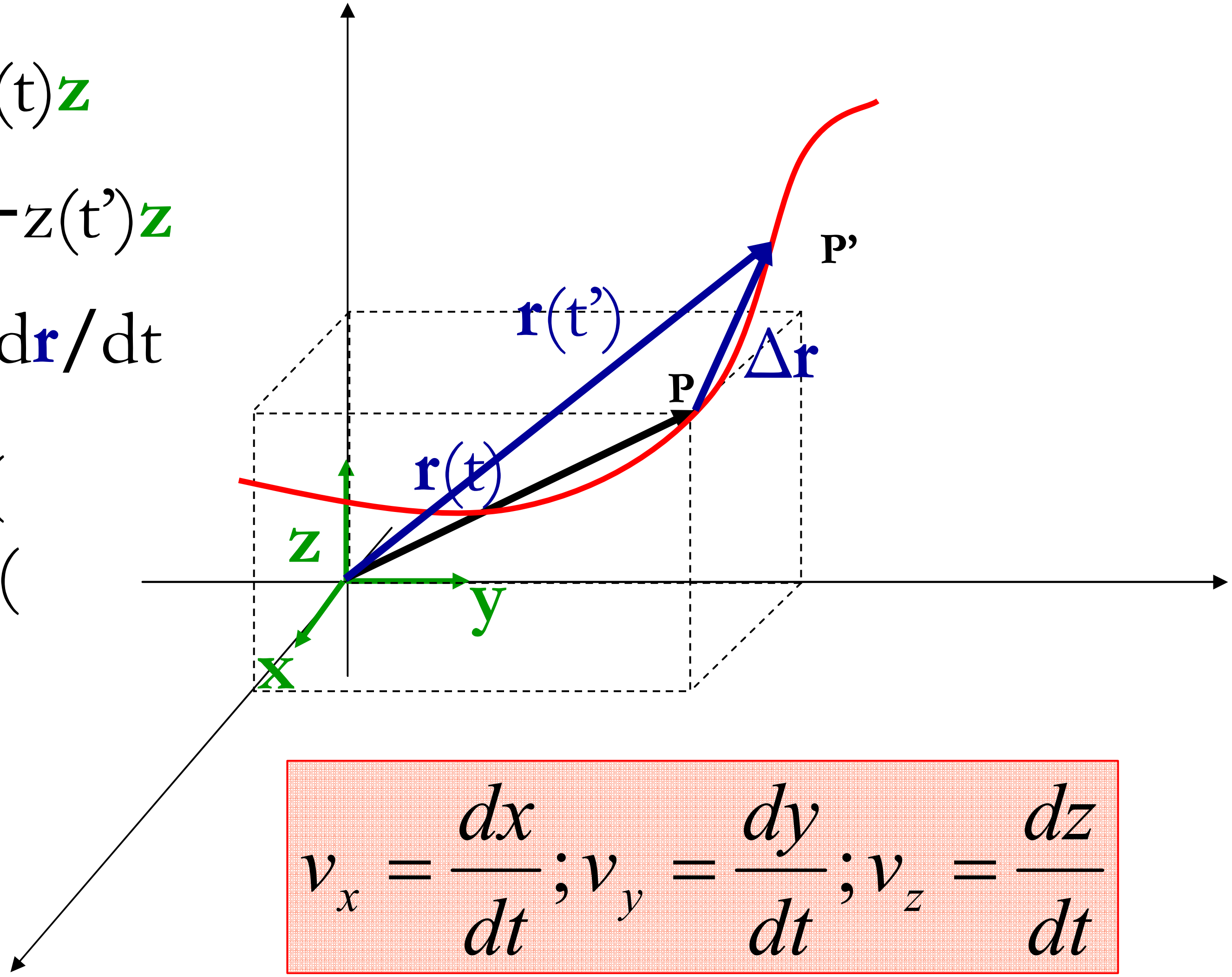
Velocity in terms of Components

$\mathbf{r}(t)=x(t)\mathbf{x}+y(t)\mathbf{y}+z(t)\mathbf{z}$

$\mathbf{r}(t')=x(t')\mathbf{x}+y(t')\mathbf{y}+z(t')\mathbf{z}$

$\mathbf{v}(t)=\lim_{\Delta t \to 0} \Delta\mathbf{r}/\Delta t = d\mathbf{r}/dt$

$=(dx(t)/dt)\mathbf{i}+(dy(t)/dt)\mathbf{j}+d(z(t)/dt)\mathbf{k}$



$\boxed{v_x\,,\ \ v_y,\ \ v_z}$

$$v_x = \frac{dx}{dt}; v_y = \frac{dy}{dt}; v_z = \frac{dz}{dt}$$

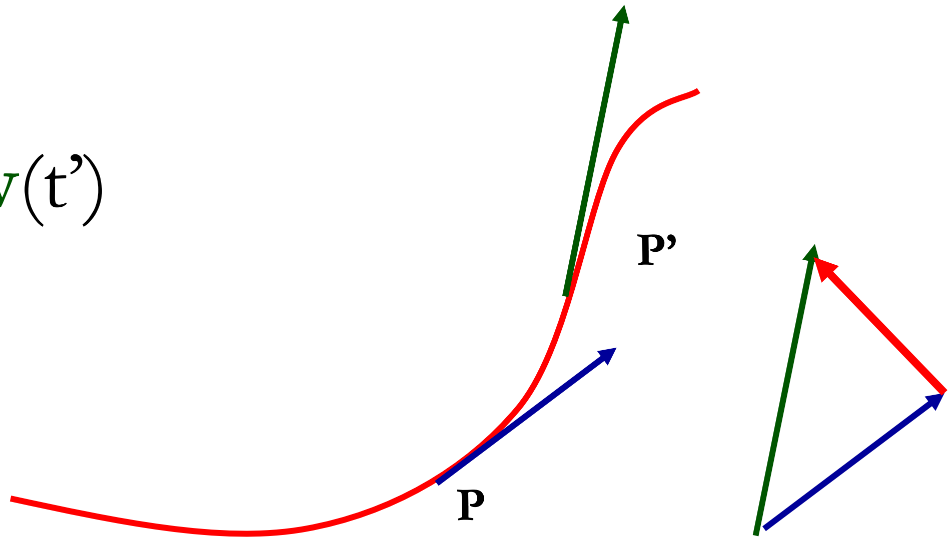
Acceleration

Velocity at time t, $\mathbf{v}(t)$

Velocity at time $t' = t + \Delta t$, $\mathbf{v}(t')$

$$\Delta \mathbf{v} = \mathbf{v}(t+\Delta t) - \mathbf{v}(t)$$

$$\mathbf{a}_m = \Delta \mathbf{v} / \Delta t$$

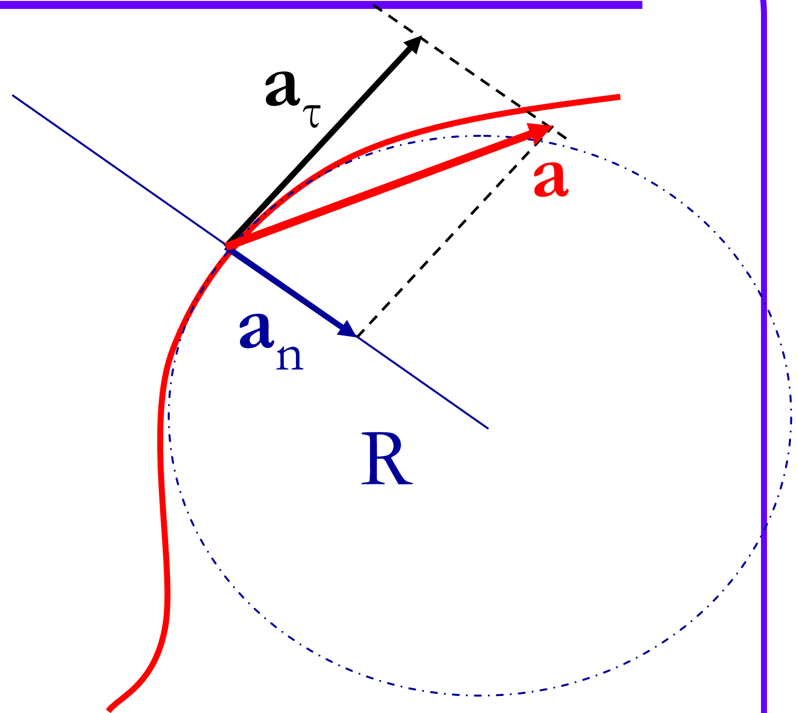


$\mathbf{a}_m$ average acceleration

$$[\mathbf{a}_m] = l^1 \cdot t^{-2} \;; \qquad m/s^2$$

Instantaneous Acceleration

$$\mathbf{a}(t)=\lim_{\Delta t \to 0} \Delta\mathbf{v}/\Delta t = d\mathbf{v}/dt$$



$$\mathbf{v}(t)= v(t)\boldsymbol{\tau}$$

$$\mathbf{a}= (dv/dt)\boldsymbol{\tau} + v\,(d\boldsymbol{\tau}/dt)$$

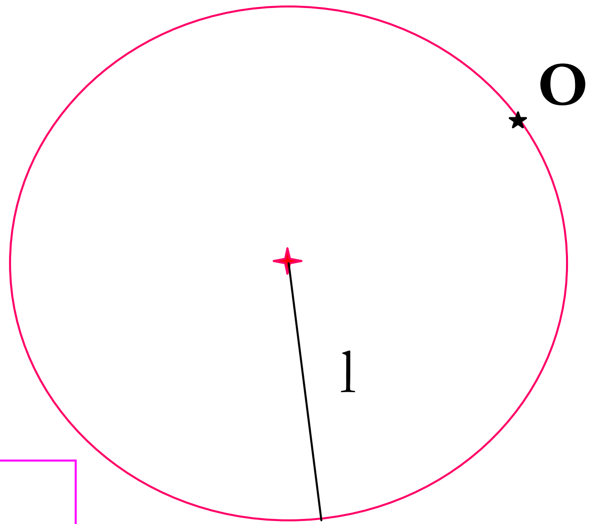
$$\mathbf{a}_\tau= (dv/dt)\boldsymbol{\tau}$$

$\mathbf{a}_\tau$ tangent acceleration

$$\mathbf{a}_n= v\,(d\boldsymbol{\tau}/dt) = (v^2/R)\mathbf{n}$$

$\mathbf{a}_n$ centripetal acceleration

Circular Motion (trajectory is a circle)



$S(t) = bt \;= (2\pi l/T)t \;= 2\pi l\nu t \;= l\omega t$

T: period, [T]=t, s

ν: frequency, [ν]=t^{-1}, Hz; ν=1/T, number of turns in unit time

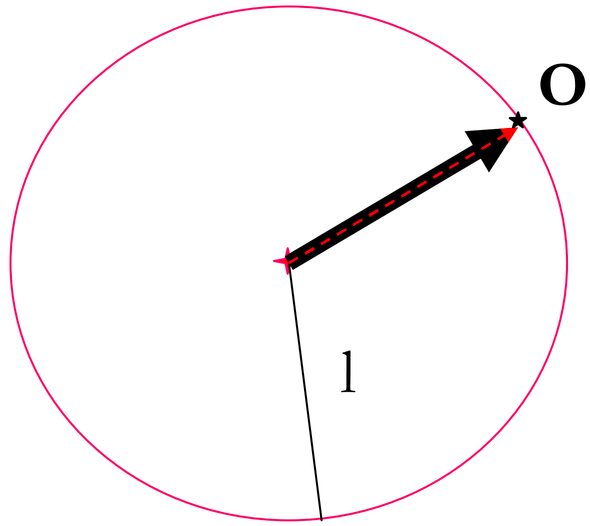
ω: angular velocity [ω]=t^{-1}, rad/s, $\omega=2\pi\nu= 2\pi/T$

Uniform Circolar Motion

AverageVelocity $\mathbf{v}_m = \Delta\mathbf{r} / \Delta t$ (which Δt ?)

- Example 1 $\Delta t = T$

$v_m = 0$



Uniform Circolar Motion

AverageVelocity $\mathbf{v}_m = \Delta\mathbf{r} / \Delta t$ (which Δt ?)

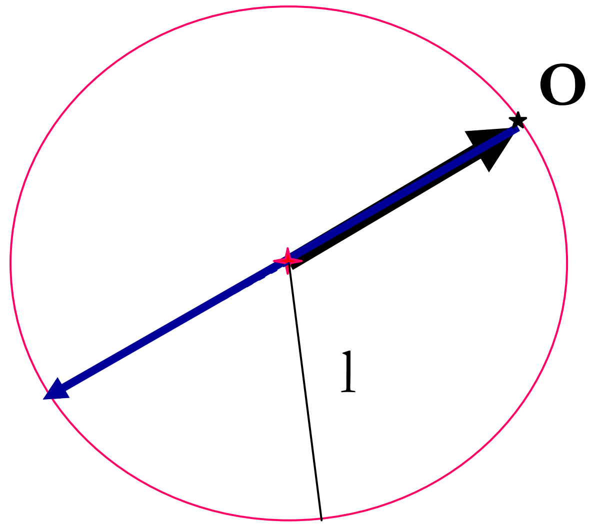
- Example 1 $\Delta t = T$

$v_m = 0$

- Example 2 $\Delta t = T/2$

Direction of $\mathbf{v}_m$

$v_m = 2l/\Delta t = 4l/T$

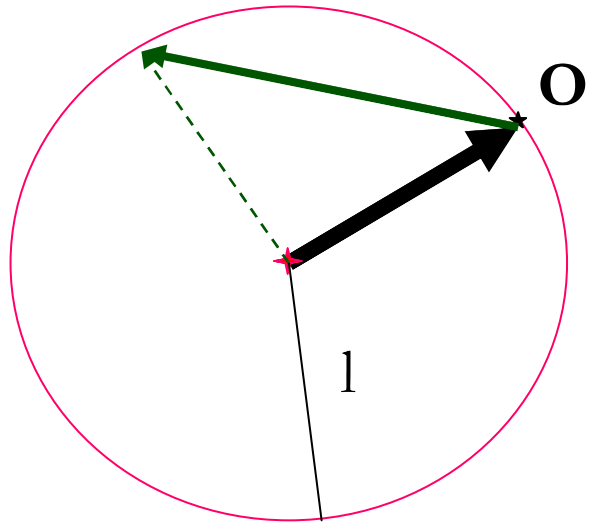


Uniform Circolar Motion

Average velocity $\mathbf{v}_m = \Delta\mathbf{r} / \Delta t$

- Ex 1 $\Delta t = T$

$v_m = 0$



- Ex 2 $\Delta t = T/2$

$v_m = 2l/\Delta t = 4l/T$

- Ex. 3 $\Delta t = T/4$

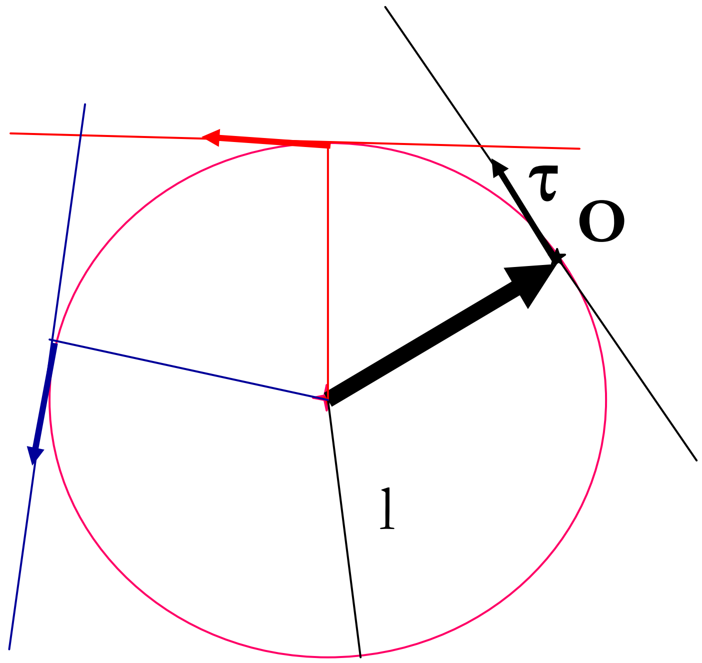
$v_m = l\sqrt{2}/\Delta t = 4l\sqrt{2}/T$

Uniform Circolar Motion

Velocity $\mathbf{v}(t)=\lim_{\Delta t \to 0} \Delta\mathbf{r}/\Delta t = d\mathbf{r}/dt$

$\mathbf{v}(t)= v(t)\,\boldsymbol{\tau} =(dS/dt)\,\boldsymbol{\tau}$

direction of **v**



module of **v**:

$S(t)= bt =2\pi lt/T=2\pi l\nu t=l\omega t$

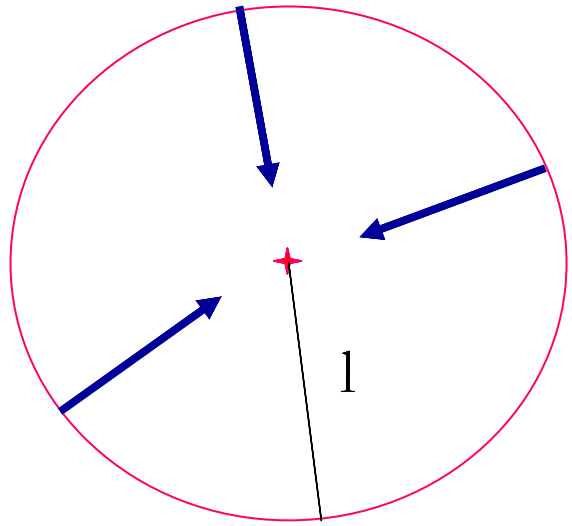
$v(t)= dS/dt= b =2\pi l/T=2\pi l\nu =l\omega$

Uniform Circolar Motion

Velocity $v(t)= 2\pi l/T=2\pi l\nu = l\omega$

$\mathbf{a}= (dv/dt)\boldsymbol{\tau} +(v^2/R)\mathbf{n}$

$\mathbf{a}_\tau= (dv/dt)\boldsymbol{\tau} =0$



$\mathbf{a}_n= (v^2/l)\mathbf{n}=4\pi^2(l/T^2)\mathbf{n} =4\pi^2 l\nu^2 =l\omega^2$

only $\mathbf{a}_n$ centripetal acceleration