Waves and Optics - PHY204 (Smaldone - Sassi)

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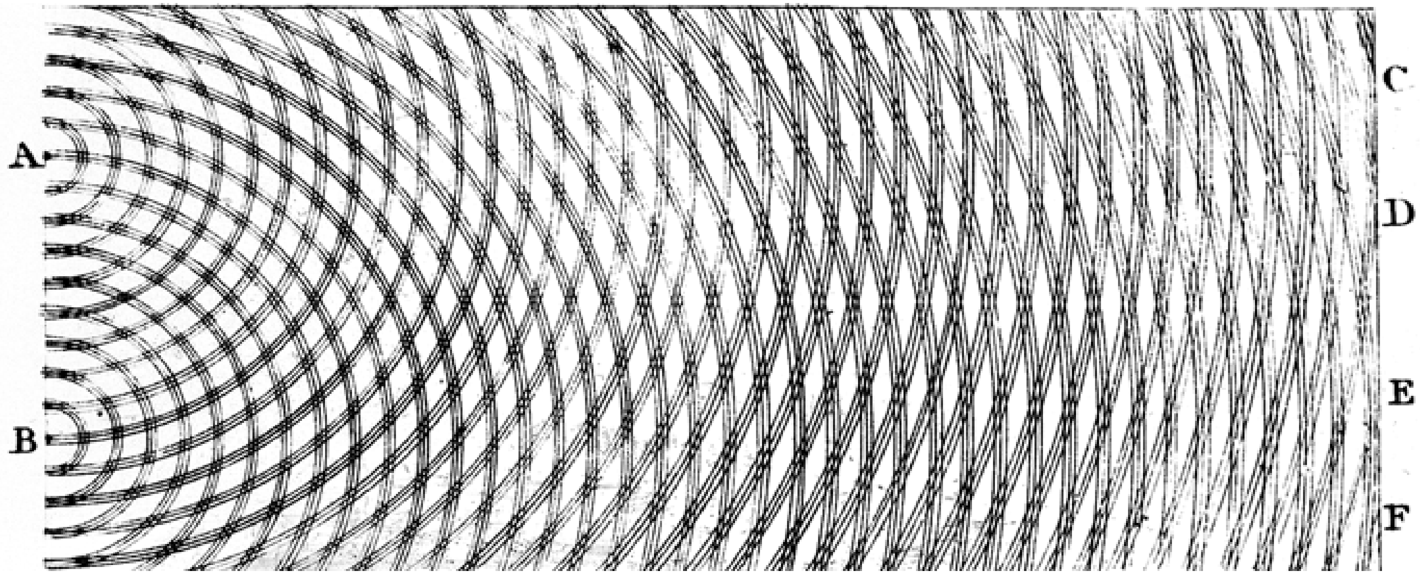
7

Interference

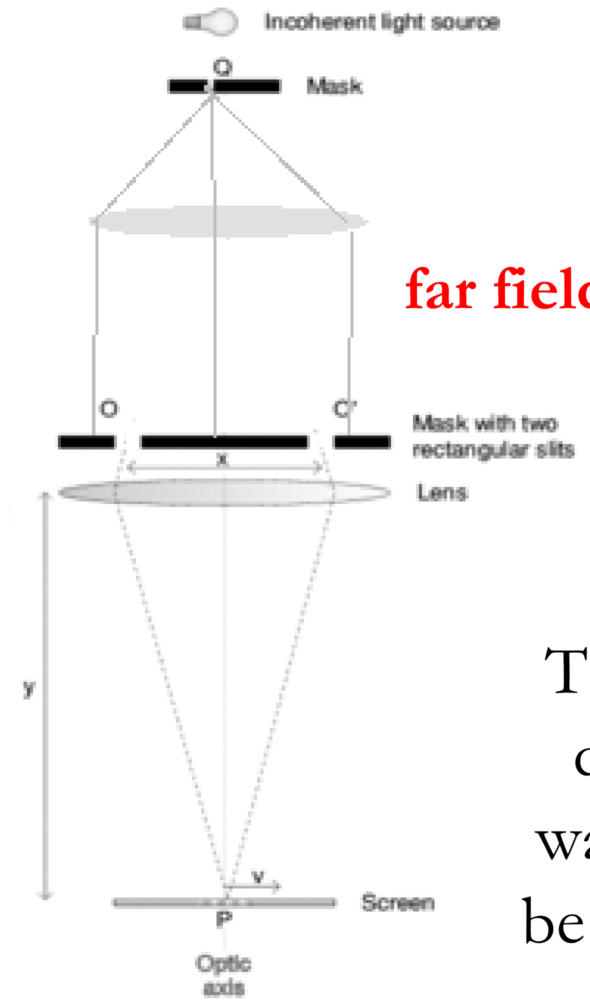
Young’s Experiment

The **double-slit** experiment have been first performed by the English scientist Thomas Young in 1801 in an attempt to resolve the question of whether light was composed of particles (Newton's "corpuscular" theory), or rather consisted of waves. The interference patterns observed in the experiment seemed to discredit the corpuscular theory, and the wave theory of light remained well accepted until the early 20th century.

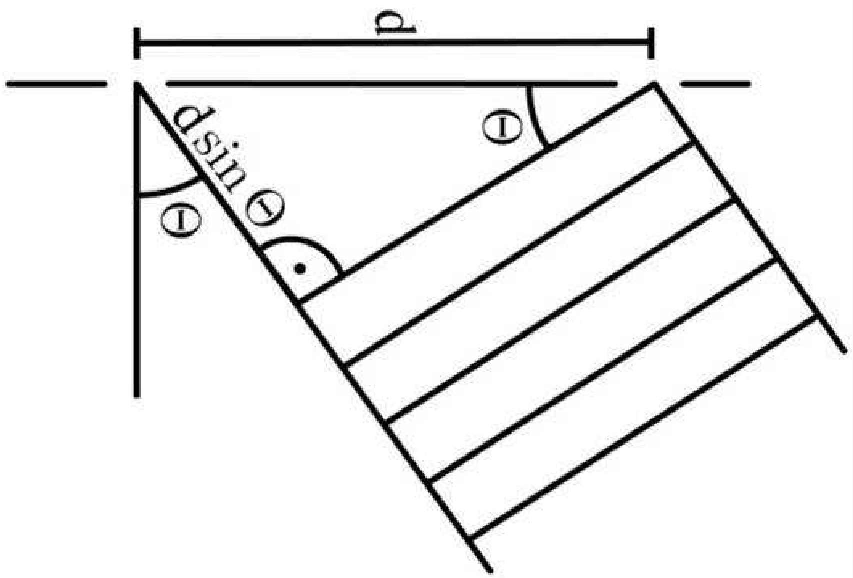
The original drawing by T. Young to illustrate its experiments.



Double-slit Experiment: Schema

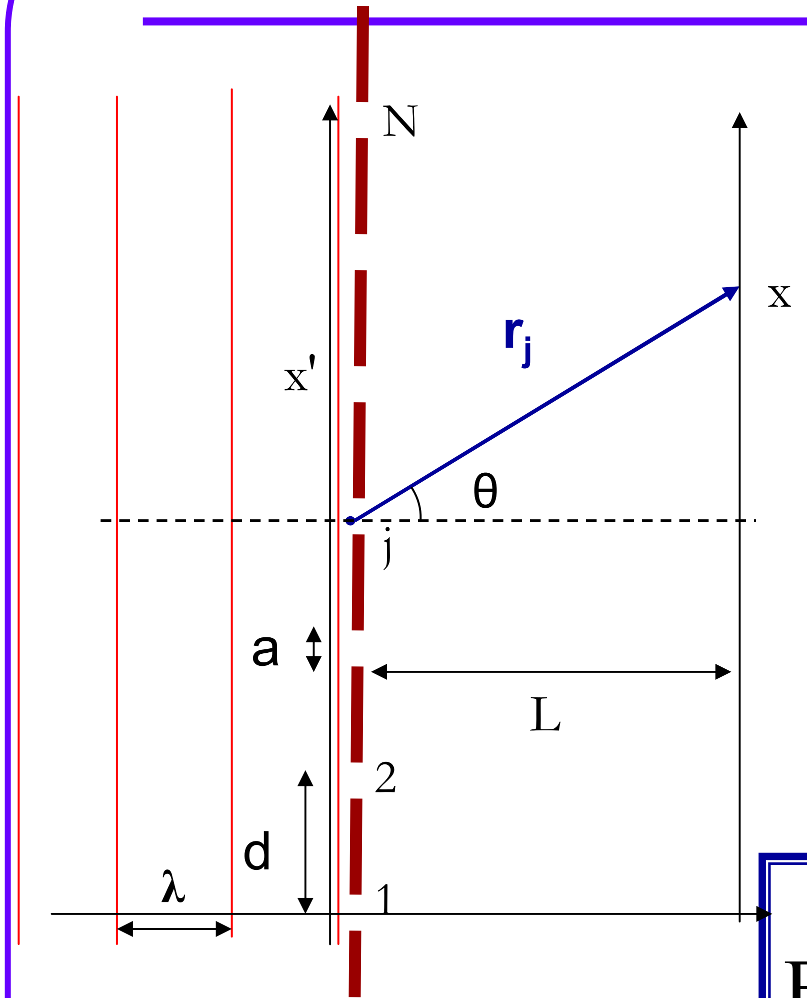


far field observations



To have a **constructive interference** along the θ direction the path length difference between the wavefronts coming from the two apertures have to be an integer number of wavelengths: $\mathbf{d \sin\theta = m\lambda}$

Quantitative Analysis of N-slits Interference



d=spacing between two slits

L=screen distance from the plane of the slits

N= total number of slits

Wave emanating from the slit #j:

$$\mathrm{E}_{\mathrm{j}}(x,t)=\frac{\mathrm{E}_0}{r_j}e^{-i\omega t}e^{ikr_j}$$

Total wave at x position on the screen:

$$\boxed{\mathrm{E}(x,t)=\sum_{j=1}^{N}\mathrm{E}_{\mathrm{j}}(x,t)=\mathrm{E}_0 e^{-i\omega t}\sum_{j=1}^{N}\frac{e^{ikr_j}}{r_j}}$$

N-slits Interference: Fraunhofer approx

$$r_j = \sqrt{(x-jd)^2 + L^2} = L\sqrt{\frac{(x-jd)^2}{L^2}+1}$$

L » | (x-jd) | **far field**:

$$r_j \approx L\left(1+\frac{(x-jd)^2}{2L^2}\right) \qquad \frac{1}{r} \approx \frac{1}{L} \qquad e^{ikr_j} \approx e^{ikL\left(1+\frac{(x-jd)^2}{2L^2}\right)} =$$

$$= e^{ikL} e^{ik\left(\frac{x^2-2xjd+j^2d^2}{2L}\right)} = e^{ikL} e^{\frac{ikx^2}{2L}} e^{-\frac{ikxjd}{L}} e^{\frac{ikj^2d^2}{2L}}$$

$$e^{\frac{ikx^2}{2L}} \approx 1 \;\therefore\; e^{\frac{ikj^2d^2}{2L}} \approx 1 \Rightarrow e^{ikr} \approx e^{ikL} e^{-\frac{ikxjd}{L}}$$

N-slits Interference: the Solution for E

$$E(x,t) = E_0 \frac{e^{ik\left(\frac{x^2}{2L}+L\right)}}{L} \sum_{j=1}^{N} e^{ik\frac{xjd}{L}}$$

The sum has the form of a geometric sum and the can be evaluated to give:

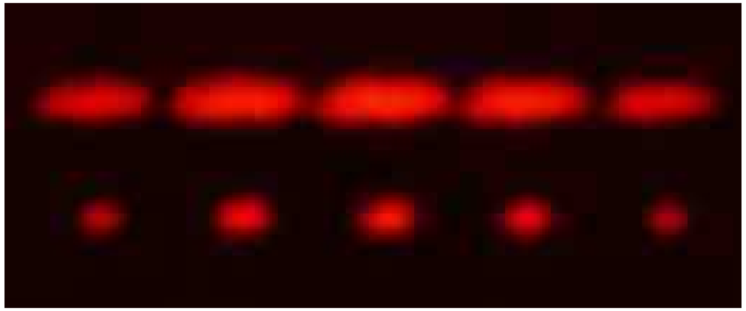
$$E(x,t) = E_0 \frac{e^{ik\left(\frac{x^2}{2L}\right)}}{L} \frac{\sin\left(\frac{2\pi Nd \sin\theta}{2\lambda}\right)}{\sin\left(\frac{2\pi d \sin\theta}{2\lambda}\right)} e^{i\frac{kxd}{2L}(N-1)}$$

(remember: $k=2\pi/\lambda$)

N-slits Interference: the Solution for I

To I(θ) is proportional to $|E|^2$, indicating as I_0 the product of all the constants, the solution is:

$$I(\theta) = I_0 \left(\frac{\sin \frac{\pi N d \sin \theta}{\lambda}}{\sin \frac{\pi d \sin \theta}{\lambda}} \right)^2$$



2 slits

5 slits

Interference of red laser light

Double-slit maxima

$$\mathrm{I}(\theta) = I_0 \left(\frac{\sin(2\pi d \sin\theta / \lambda)}{\sin(\pi d \sin\theta / \lambda)} \right)^2$$

Maxima $\Rightarrow$ when denominator=0

$$\boxed{\frac{\pi d \sin\theta}{\lambda} = n\pi \Rightarrow \sin\theta = n\frac{\lambda}{d}}$$

n is the **fringe order**

- ***n*** is a positive o negative integer
- there is a $\mathbf{n_{max}}$ ($\mathbf{n_{max}}$=max integer $\leq d/\lambda$)
- total number of fringes $=2\mathbf{n_{max}}+1$ (from $-\mathbf{n_{max}}$ to $+\mathbf{n_{max}}$)

Double slit exercises

Using a double slit spaced 5 μm, the 1st order fringe is at 7.20^0, figure out the wavelength of the used laser. Which is the total number of visible fringes? [629 nm, 15]

Using a laser at λ=534 nm, the 3rd order fringe is at 30.80^0 in a double slit experiment, determine the spacing between the slits. Which is the total number of visible fringes? [3.3 μm, 11]

5-slits versus double slit

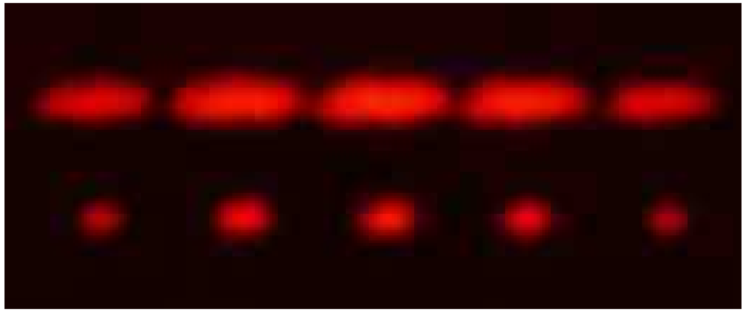
$$I(\theta) = I_0 \left(\frac{\sin(5\pi d \sin\theta / \lambda)}{\sin(\pi d \sin\theta / \lambda)} \right)^2$$

Maxima 5-slit ⇒ when denominator=0

$$\boxed{\frac{\pi d \sin\theta}{\lambda} = n\pi \Rightarrow \sin\theta = n\frac{\lambda}{d}}$$

same as 2-slit!!!

only the **fringe width** is narrower to respect 2-slit (the fringe width is proportional to the numerator period!)



2 slits

5 slits

Interference of red laser light