Waves and Optics - PHY204
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Waves

Wave Motion

Waves are everywhere:
Earthquakes, vibrating strings of a guitar, light from the sun; a wave of heat, of bad luck, of madness…

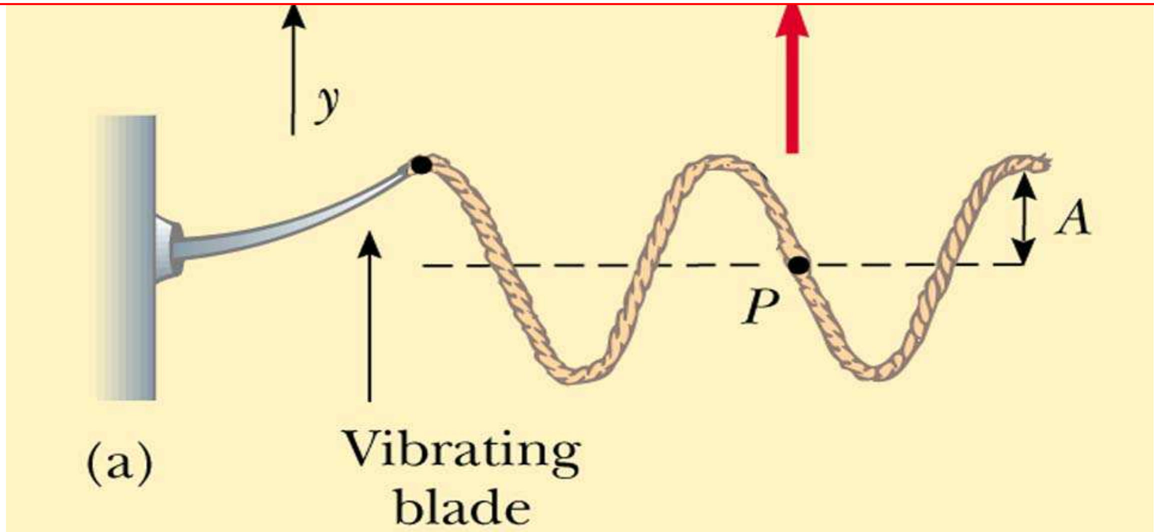
Something moving, passing by, bringing a change and then going away, sometimes without a trace…

Some waves are man-made:
radio waves, stadium waves, microwaves, annoying sound waves of a physics lecture, rap music blaring out of an audio system in a car…

Waves Appl.

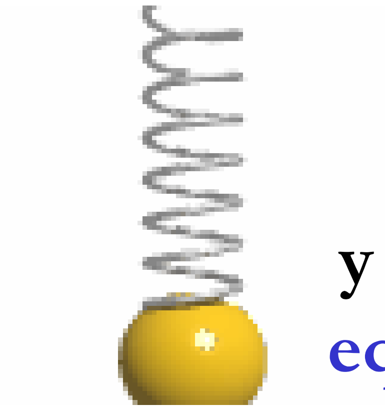
Wave Definition

A wave is a traveling disturbance that transports energy but not matter.



- Mechanical waves require
  - Some source of disturbance
  - A medium that can be disturbed
  - Some physical connection between or mechanism though which adjacent portions of the medium influence each other
- All waves carry energy and momentum

Harmonic oscillation & sinusoidal motion



$$y(t) = A\sin(\omega t + \varphi) = A\sin(2\pi f t + \varphi)$$

y - height of the object with respect to its equilibrium position;

A -amplitude of the oscillations;

$\omega = 2\pi f$ -angular frequency, measured in **rad/s**;

$f = 1/T$ -regular frequency, measured in Hertz (cycles per second) or s^{-1};

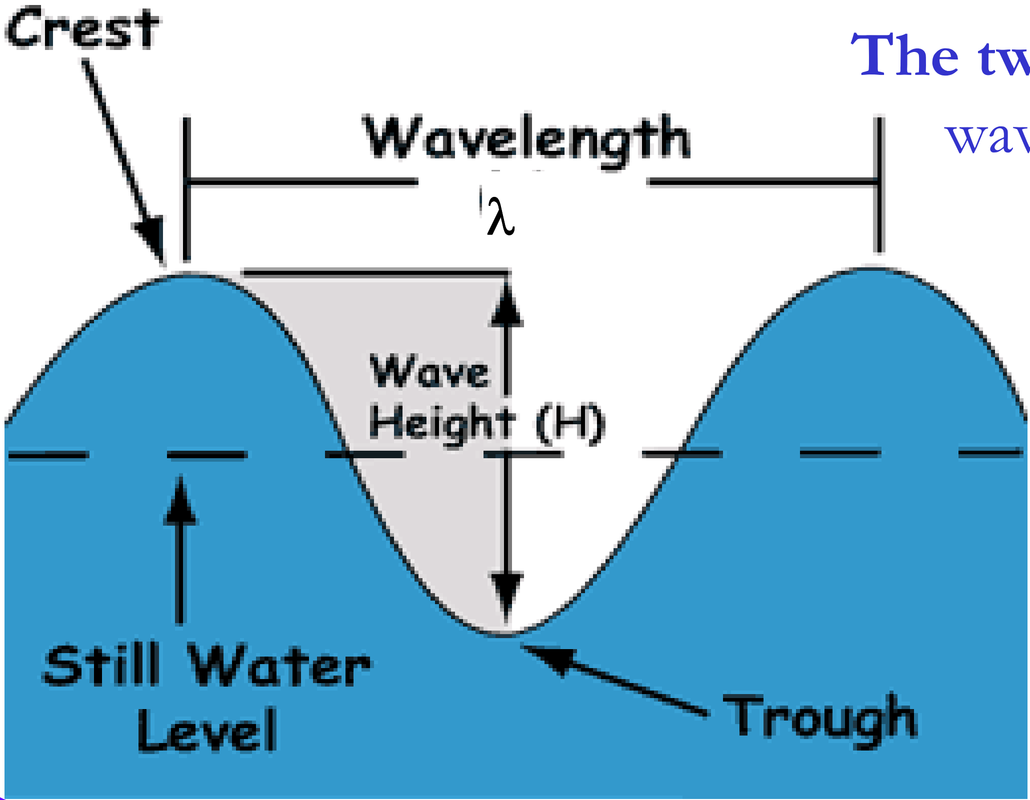
T -the period of oscillations in seconds

Wavelength

Waves have all the same stuff as the oscillation do: amplitude, frequency, energy…

But they also have much more, because they propagate in space…

More to think about… ☺

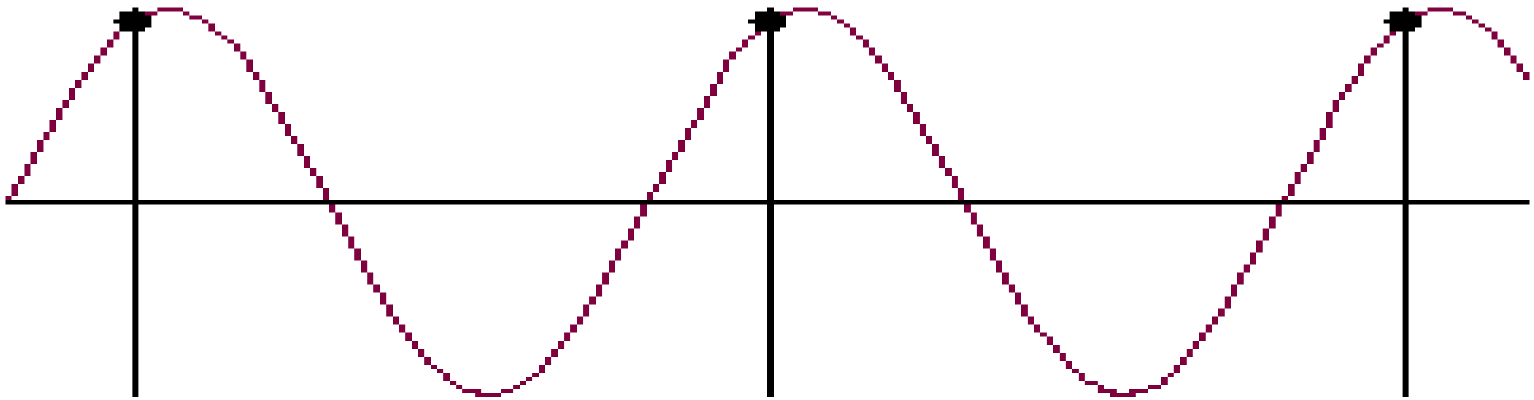


The two basic new parameters are:

wave length **and** wave speed**.**

How do we calculate the speed of a traveling wave?

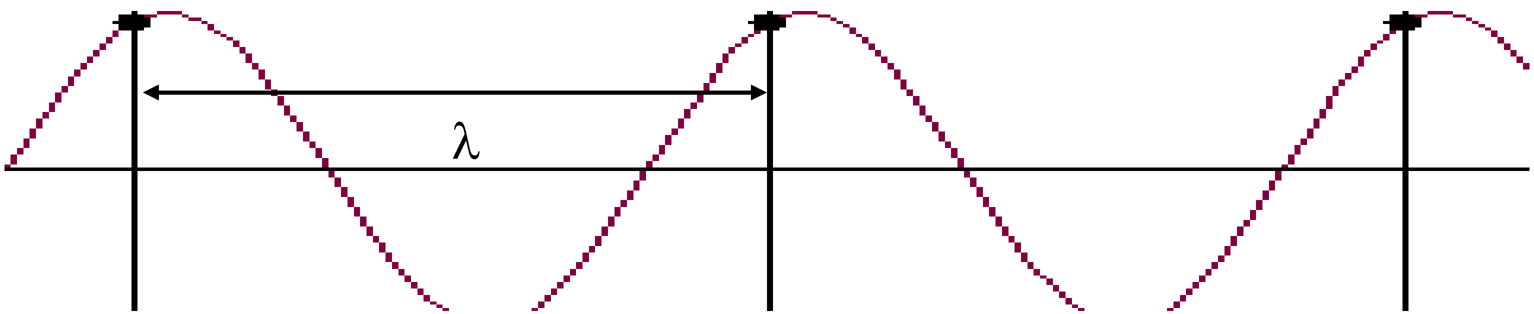
Consider polls that are placed a wavelength λ **apart.**
The oscillations there are always in phase**.**
The time it takes a crest to travel between them is the period of oscillations T **(**exactly the time between two consecutive crests at a single pole**!).**



Therefore the wave speed, v, can be calculated as.

$$\boxed{v = \frac{\lambda}{T} = \lambda f}$$

Wave motion (a traveling wave)



There is NO **direct connection between the** wave speed**:**

$$v = \frac{\lambda}{T}$$

and the velocity of motion of material particles:

$$v_y(t) = \frac{dy}{dt} = \omega A \cos(\omega t + \varphi)$$

How do we calculate the frequency of a traveling wave?

$$\boxed{\mathrm{v} = \frac{\lambda}{T} = \lambda f}$$

Blue light has shorter wavelength than red light; what about their frequencies?

$$f = \frac{\mathrm{v}}{\lambda}$$

Sound wave and light wave with the same wavelength. Which has the higher frequency?

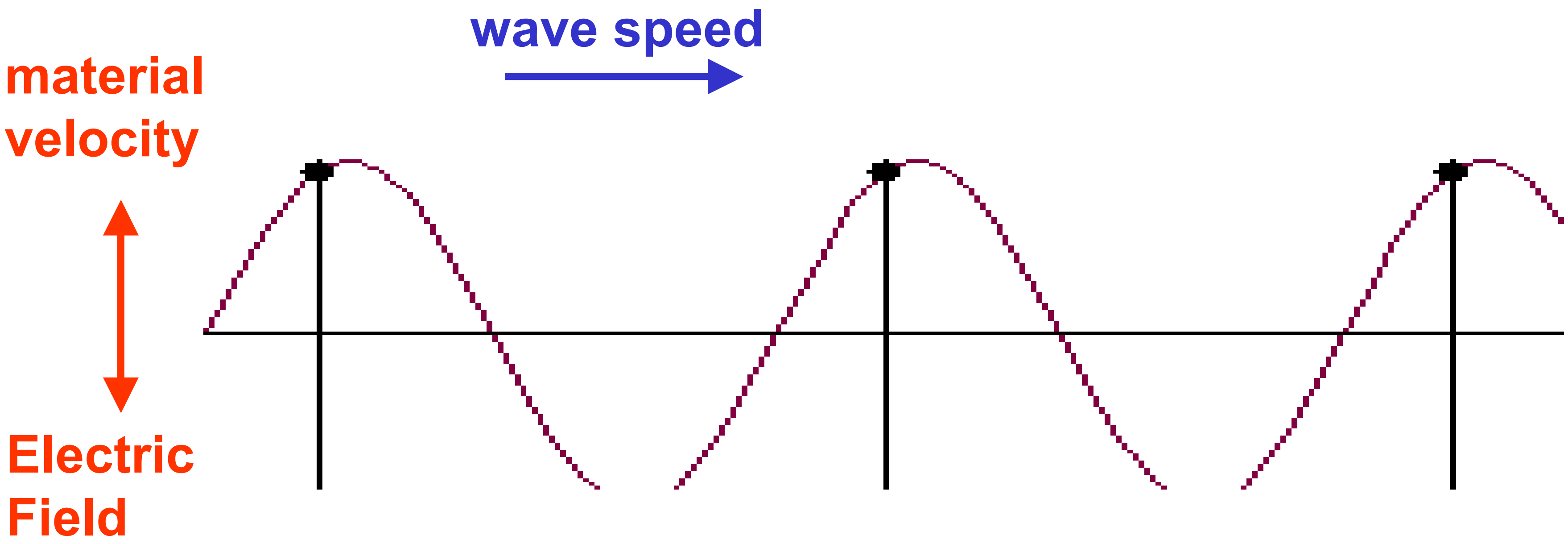
Harmonic Waves:

They are sinusoidal waves in wich the crests move with a constant speed, while the material elements oscillate harmonically.

$$A\sin(2\pi f t + \varphi \pm .....) \quad \text{or} \quad A\cos(2\pi f t + \vartheta \pm .....)$$

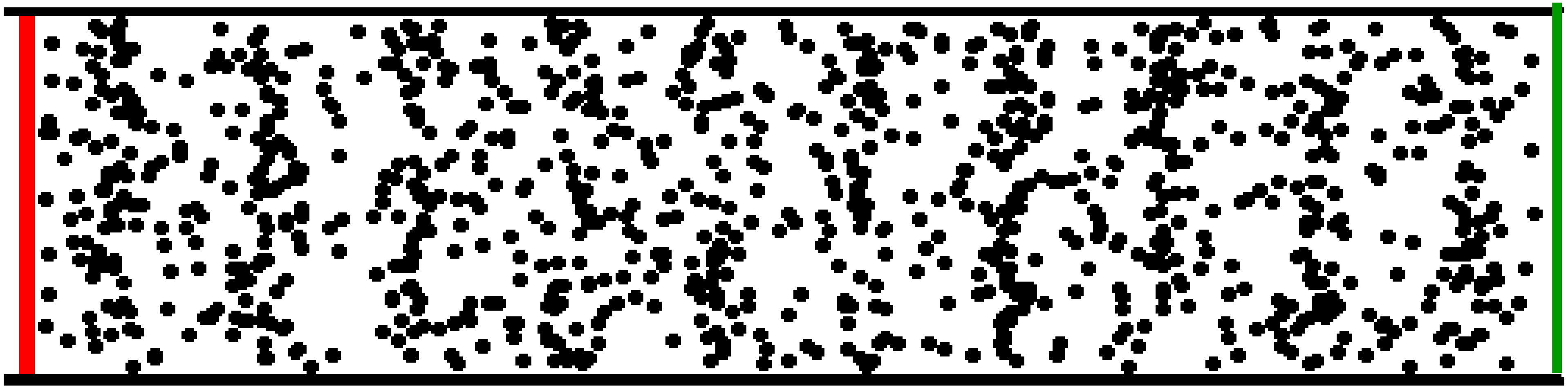
What is missing here ??

Transverse waves



Transverse wave **– material elements (medium) move (or variable Electric Field is) perpendicularly of the direction of wave propagation**

Longitudinal Waves



In a longitudinal wave the particle displacement is parallel to the direction of wave propagation.

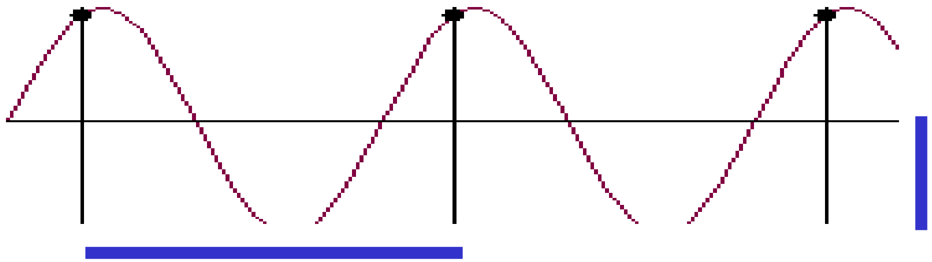
The animation above shows a one-dimensional longitudinal plane wave propagating down a tube.

The particles **do not move down the tube** with the wave; they simply oscillate back and forth about their individual equilibrium positions. Pick a single particle and watch its motion!

The wave is seen as the motion of the compressed region (i.e. it is a pressure wave), which moves from left to right.

Transverse vs. longitudinal wave

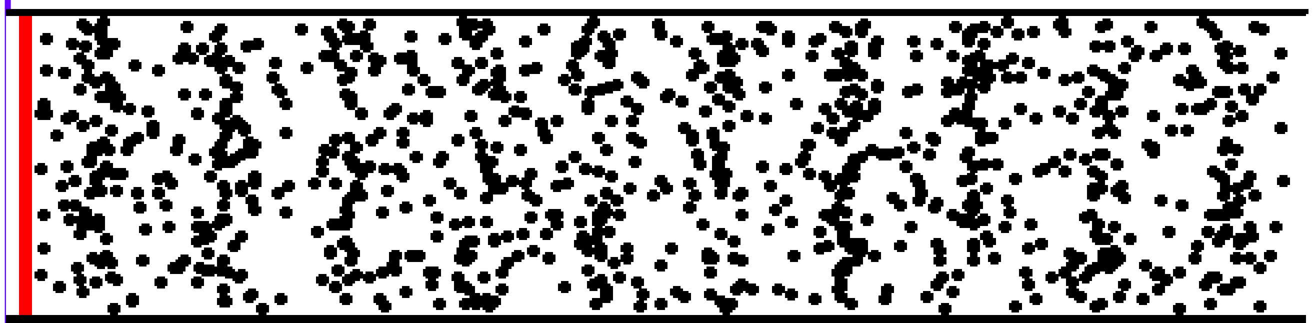
Both propagate from left to right, but cause disturbances in different directions, Δy and Δx.



wavelength, λ

$$\Delta y(t) = A_y \cos \omega t$$

A_y - amplitude

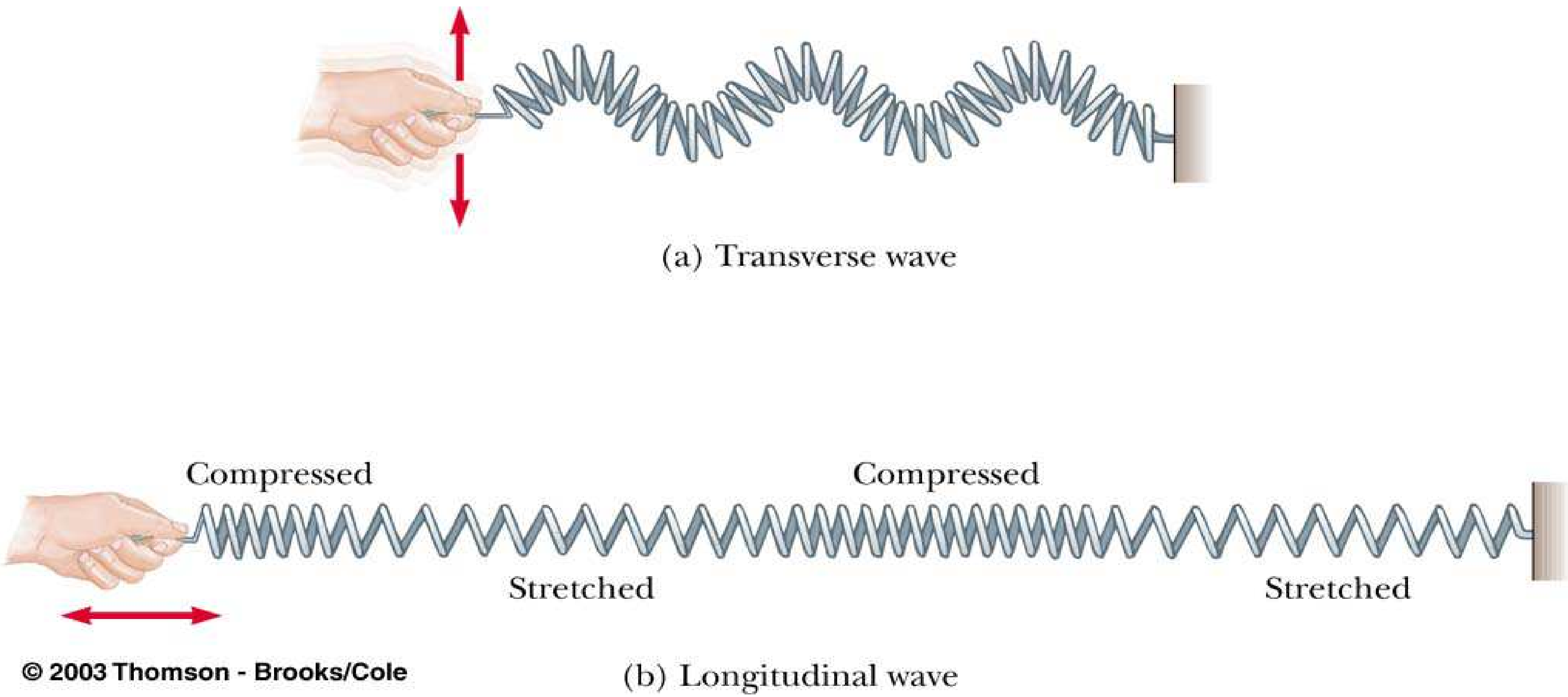


wavelength, λ

$$\Delta x(t) = A_x \cos(\omega t)$$

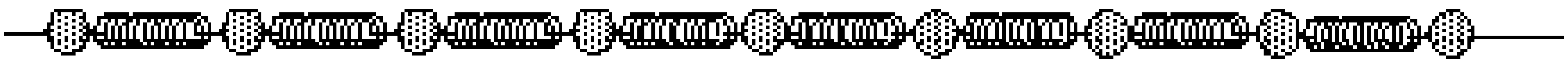
A_x - amplitude

Waves on a Spring



Harmonic waves are not the only possible type of waves!

A wave can also have a shape of a propagating pulse.
True for both transverse and longitudinal **waves.**



Wave train

A harmonic wave and a pulse are extreme cases.
The intermediate case is a wave train – a finite duration sinusoidal.

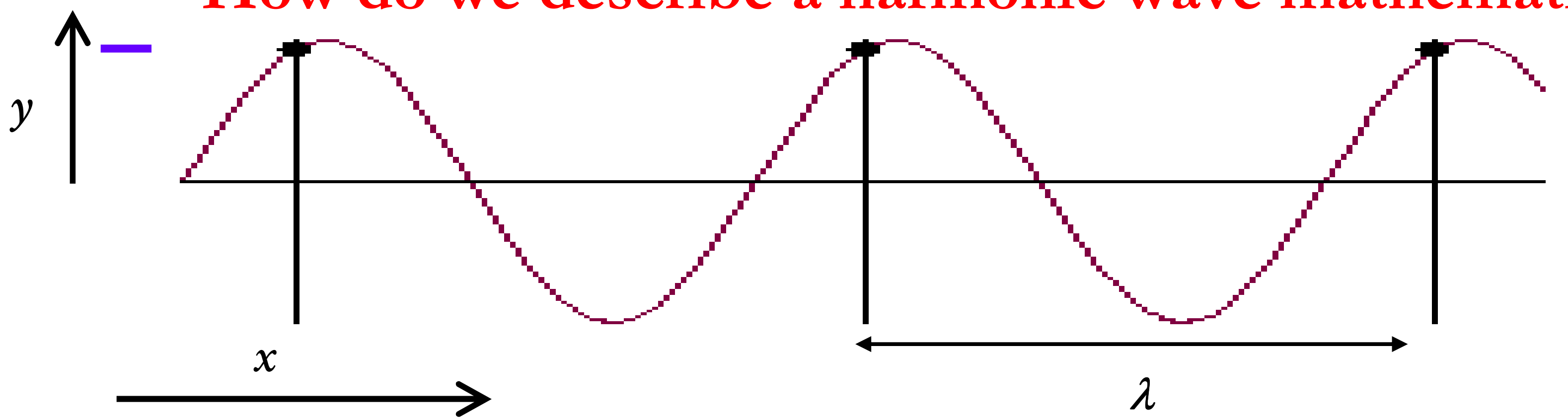
(*a*)

(*b*)

(*c*)

FIGURE 16-6 (*a*) A pulse, (*b*) a continuous wave, and (*c*) a wave train.

How do we describe a harmonic wave mathematically?



• Features to incorporate:

in any point in space the wave produces harmonic oscillations of a type:

$y(x)=A_y\cos(\omega t+\varphi)$ - ω angular frequency - φ initial phase

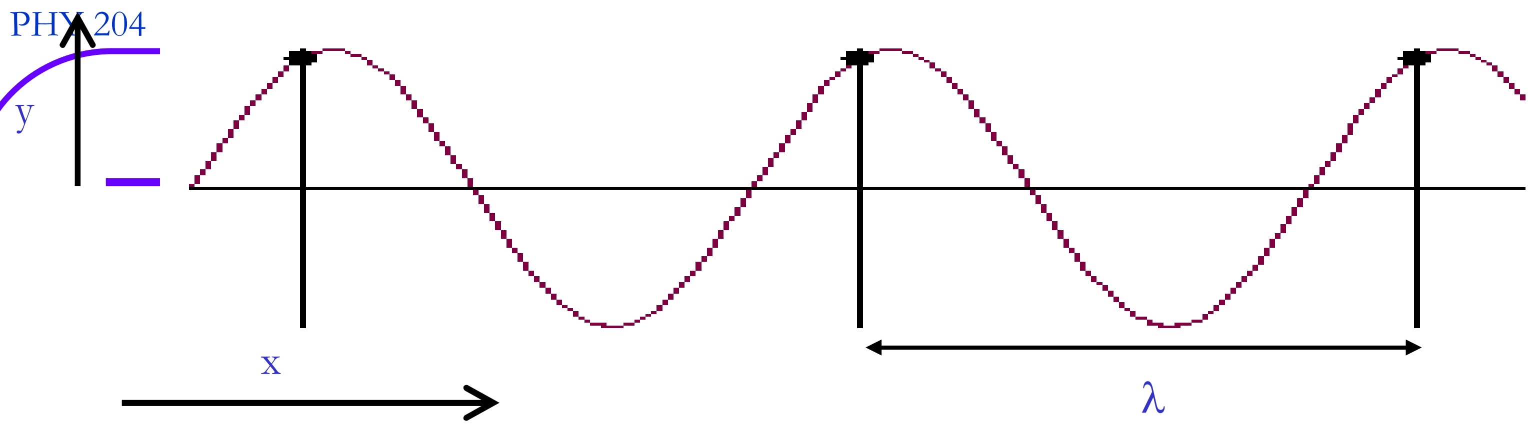
if we "freeze" the wave in time, we will see a harmonic function in space:

$y(x)=A_y\cos(kx+\varphi)$ what is this k ?

if we freeze the wave and move 1 wavelength λ along it, we are supposed to see the same level of disturbance **y**

Therefore, it must be $k\lambda=2\pi$ so that

$$y(x+\lambda) = A_y \cos(k(x+\lambda)+\varphi) = A_y \cos(kx+2\pi+\varphi) = y(x)$$



$$y(t) = A_y \cos(\omega t + \varphi) \qquad y(x) = A_y \cos(kx + \varphi)$$

ω - angular frequency $\qquad \varphi$ - phase $\qquad k\lambda = 2\pi \Rightarrow k = 2\pi / \lambda$

k is measured in m^{-1}. What is the meaning of it?

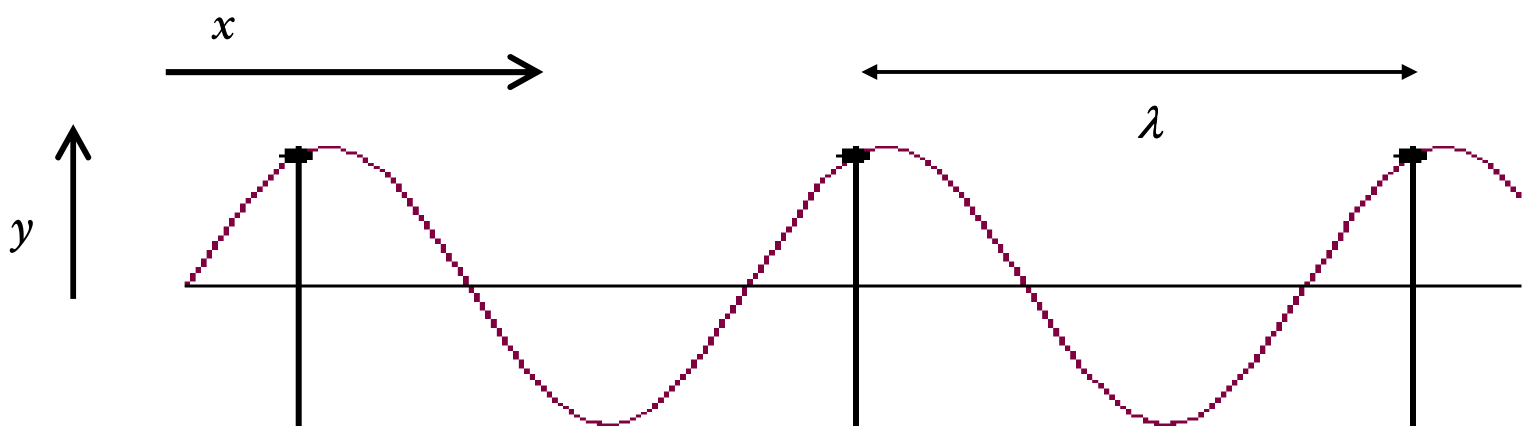
$k / 2\pi$ tells us every how many times per meter it is going to happen

$\omega / 2\pi = f$ tells us every how many times per second we are going to see a crest if we stay frozen and let the wave propagate

k is pretty much the same for space as ω is for time!

k behaves like a spatial frequency and is usually called the "wave number"

Waves: Space and Time



$$y(t) = A_y \cos(\omega t + \varphi) \qquad y(x) = A_y \cos(kx + \varphi)$$

$$\omega = 2\pi / T \qquad k = 2\pi / \lambda$$

T is period in time $\qquad$ λ is period in space

How do we unite the two equations (in time and in space)?

$$\boxed{y(x,t) = A_y \cos(kx - \omega t)}$$

Still Time and Space

$$\boxed{y(x,t) = A_y \cos(kx - \omega t)}$$

Considering only one point in space, x_0, means taking $\varphi = kx_0$

$$y(x_0,t) = A_y \cos(\varphi - \omega t) = A_y \cos(\omega t - \varphi)$$

Freezing it in time, t_0 means taking $\varphi = -\omega t_0$

$$y(x,t_0) = A_y \cos(kx + \varphi)$$

$$\omega = 2\pi / T = 2\pi f$$

$$k = 2\pi / \lambda$$

$$\boxed{\lambda = \frac{\mathrm{v}}{f} \; ; \; k = \frac{2\pi f}{\mathrm{v}} = \frac{\omega}{\mathrm{v}} \Rightarrow \omega = k\mathrm{v}}$$

$$y(x,t) = A_y \cos(kx - \omega t) = A_y \cos(kx - k\mathrm{v}t) = A_y \cos[k(x - \mathrm{v}t)]$$

A crest corresponds to a point, where $\boxed{k(x - \mathrm{v}t) = 0}$

Therefore position of the crest is given by $x = \mathrm{v}t$

It is moving with wave speed v !!!

Harmonic Waves Summary

$$\boxed{y(x,t) = A_y \cos(kx - \omega t)}$$ - equation of a harmonic wave

$$y(x,t) = A_y \cos[k(x - \mathrm{v}t)]$$ - the same equation rewritten in a form emphasizing propagation and wave speed

$$y(x,t) = A_y \cos[k(x + \mathrm{v}t)]$$ - what would this one stand for?

-v is changed to +v , which means that the wave is propagating in the negative *x*-direction, from right to left

In this case location of a crest is given by

$$\cos[k(x + \mathrm{v}t)] = 1$$

$$x + \mathrm{v}t = 0 \quad \Rightarrow \quad x = -\mathrm{v}t$$

How can we describe a pulse? (not a harmonic wave)

Generic Form for a One Dimensional Wave

Generic wave traveling in positive x-direction with wave speed **v**:

$$\boxed{y(x,t) = f(x - \mathrm{v}t)}$$

Here f(**x**) can be ANY function. The type of the function f(**x**) specifies the shape of the wave.

How do we know it is a propagating (traveling) wave?

y (the disturbance) depends on **x** and **t** in a VERY SPECIAL WAY: it only depends on **x-vt**

Therefore the disturbance **y** is the same as long as **x-vt** is constant, say $\mathbf{x_0}$

$$x - \mathrm{v}t = x_0 \quad \Rightarrow \quad x = \mathrm{v}t + x_0$$

A point of constant **y** disturbance (crest, trough, etc.) moves at constant wave speed, **v**

An Example

Ripples on a puddle are propagating at *34cm/s* with a frequency of *5.2Hz*.

(a) What is the period? $T = 1/f = 1/5.2 = .19\sec$

(b) What is the wavelength? $\lambda = v/f = (34cm/\sec)/5.2Hz = 6.5cm$

(c) What is the angular frequency? $\omega = 2\pi f = 10.4\pi = 32.7rad/\sec$

(d) What is the wave number? $k = 2\pi/\lambda = 2\pi/6.54 = .96cm^{-1}$

(e) Find the angular frequency from the velocity and wave number.

$$\omega = vk = 34cm/\sec\ .96cm^{-1} = 32.7rad/\sec$$

Another Example

Consider a wave whose displacement is given by *y=1.3cos(.69x+.31t).*
X and *y* are measured in centimeters and *t* in seconds.

(a) What is the period? $2\pi / .31 = 20.27 \sec$

(b) What is the wavelength? $\lambda = 2\pi/.69 = 9.1cm$

(c) What is the amplitude? $A = 1.3cm$

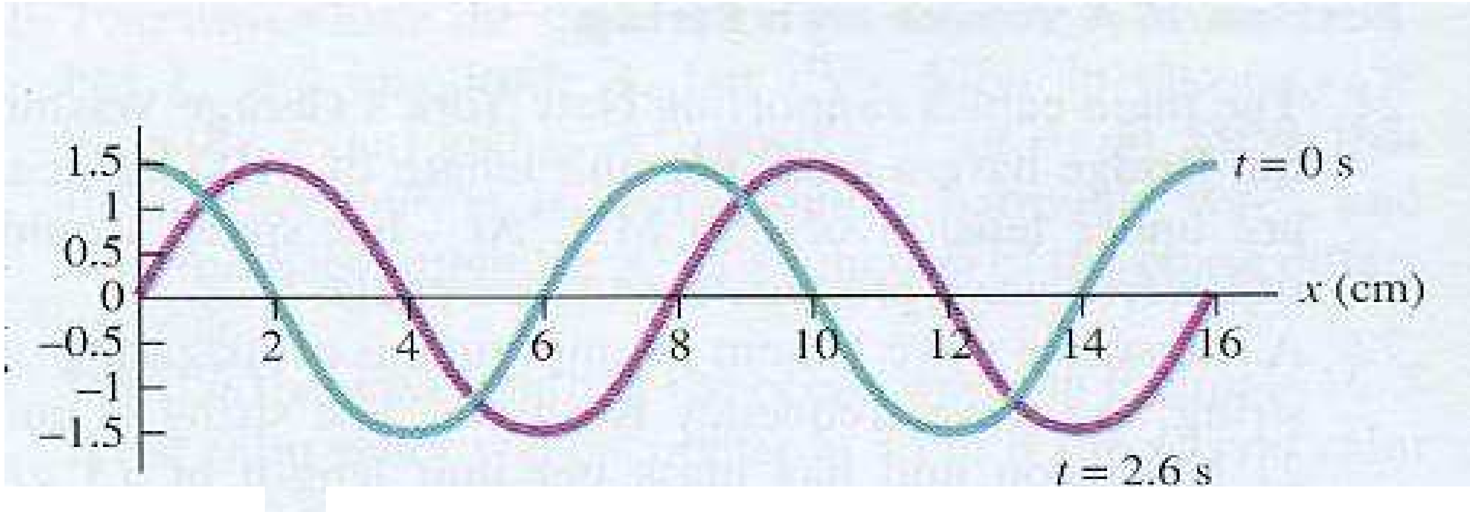
(d) What is the velocity? $-v = \lambda/T = \frac{2\pi}{.69}\frac{.31}{2\pi} = \frac{.31}{.69} =.45\frac{cm}{\sec}$

(e) What is the angular frequency and the wave number?

$$\omega = 2\pi f = 2\pi/T =.31 rad/\sec,\ k = 2\pi/\lambda =.69cm^{-1}$$

Yet Another Example

The figure shows a simple harmonic wave at *t=0*, and later at *t=2.6sec*. Write a mathematical description of the wave.



(i) $\lambda = 8cm$ (ii) $v = 2cm/2.6\sec = .77cm/\sec$

(iii) $k = 2\pi/\lambda = .785cm^{-1}, \ \omega = vk = .6rad/\sec$

$$y(x,t) = 1.5\cos(kx - \omega t) = 1.5\cos(.785x - .60t)$$

Differential Wave Equation

We can show that the general form of **one dimensional wave** is solution of **differential wave equation**:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{\mathrm{v}^2}\frac{\partial^2 f}{\partial t^2}$$

Home Work Wave -1

Proof that the harmonic wave **y(x,t)=Acos[k(x±vt)]** is solution of differential wave equation

Three Dimensional Differential Wave Equation:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \frac{1}{\mathrm{v}^2}\frac{\partial^2 f}{\partial t^2}$$

or in a more compact form (Laplacian operator)

$$\nabla^2 f = \frac{1}{\mathrm{v}^2}\frac{\partial^2 f}{\partial t^2}$$

Superposition Principle

If $E_1(x,t)$ and $E_2(x,t)$ are solutions to the wave equation, then $aE_1(x,t) + bE_2(x,t)$ is also a solution whatever a and b are

Home Work Wave -2: Proof the Superposition Principle

This means that waves (and light beams!) can pass through each other.
It also means that waves can constructively or destructively interfere.