Waves and Optics - PHY204
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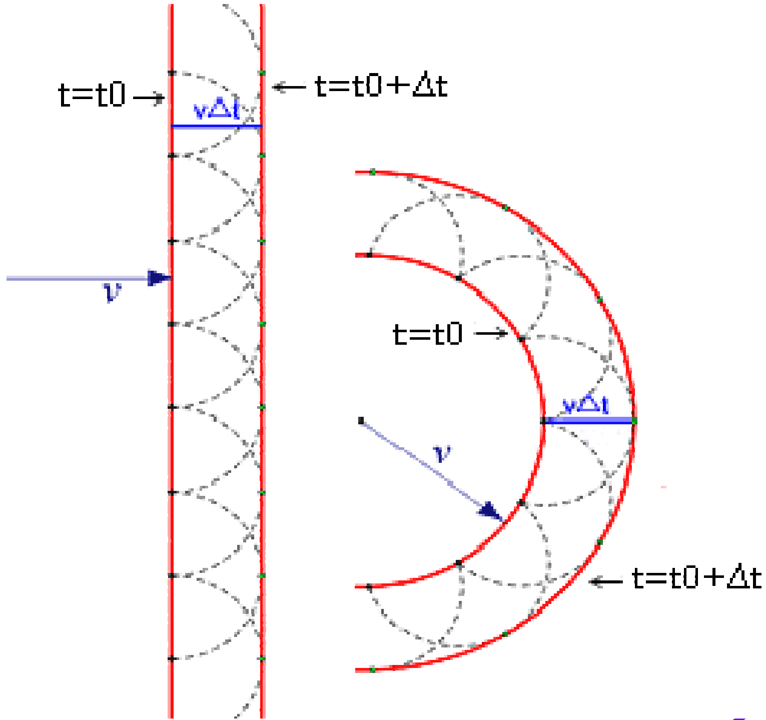
Diffraction

Huygens(–Fresnel) principle

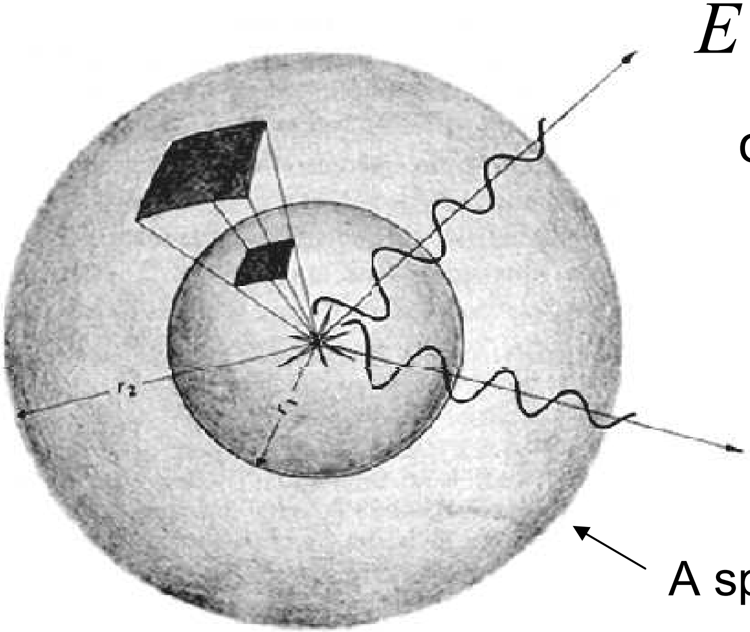
The **Huygens principle** is a method of analysis applied to problems of wave propagation. It recognizes that **each point** of an advancing wave front is in fact the **center of a fresh disturbance** and the source of a new train of waves; and that the **advancing wave** as a whole may be regarded as the **sum of all the secondary spherical waves arising from points** in the medium already traversed.

This view of wave propagation helps better understand a variety of wave phenomena, such as **reflection, refraction, diffraction**…



Spherical Waves

A spherical wave is also a solution to Maxwell's equations and is the model we need to correctly apply Huygens principle .



$$E(\vec{r},t) \propto \left(E_0 / r\right)\mathrm{Re}\{\exp[i(kr - \omega t)]\}$$

or

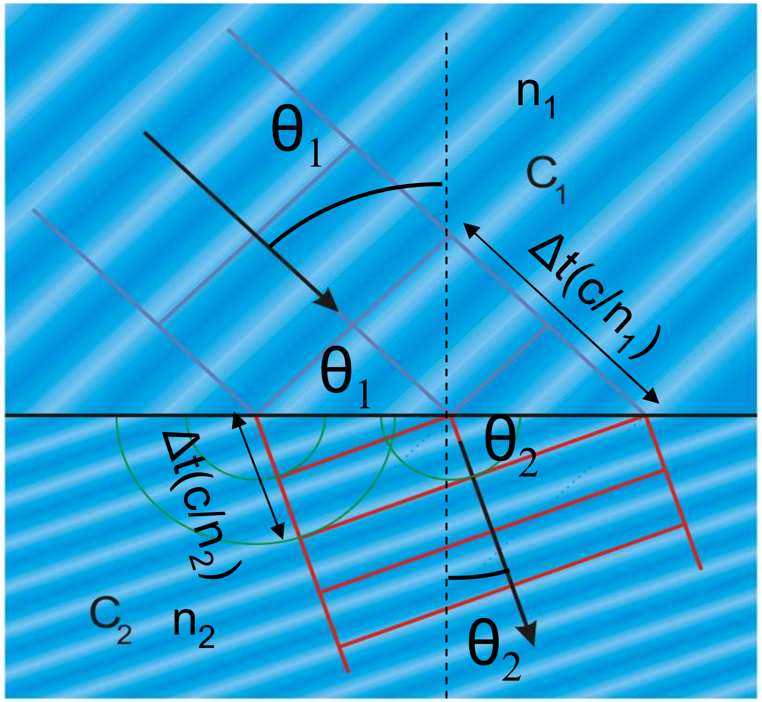
$$E(r,t) \propto \frac{E_0}{r}\cos(kr - \omega t)$$

- where k is the wavenumber=$2\pi/\lambda$
- r is the radial magnitude.

A spherical wave has spherical wave-fronts.

Unlike a **plane wave**, whose amplitude **remains constant** as it propagates, a **spherical wave weakens**. Its irradiance goes as $1/r^2$.

Huygens principle and Snell's Law



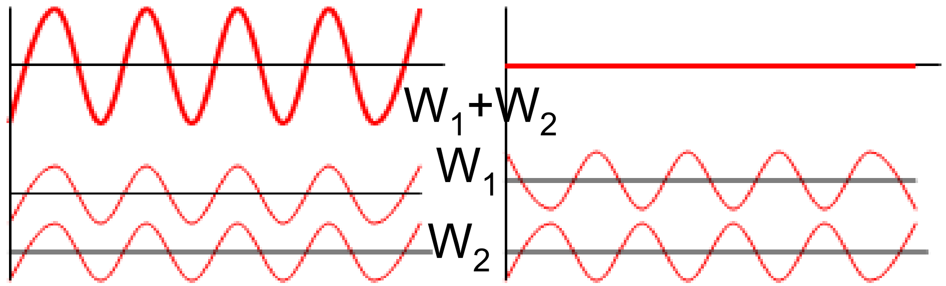
Ipotenuse = $$\frac{1}{\sin\theta_1}\frac{c\Delta t}{n_1} = \frac{1}{\sin\theta_2}\frac{c\Delta t}{n_2} \Rightarrow$$

$$\boxed{n_2 \sin\theta_2 = n_1 \sin\theta_1}$$

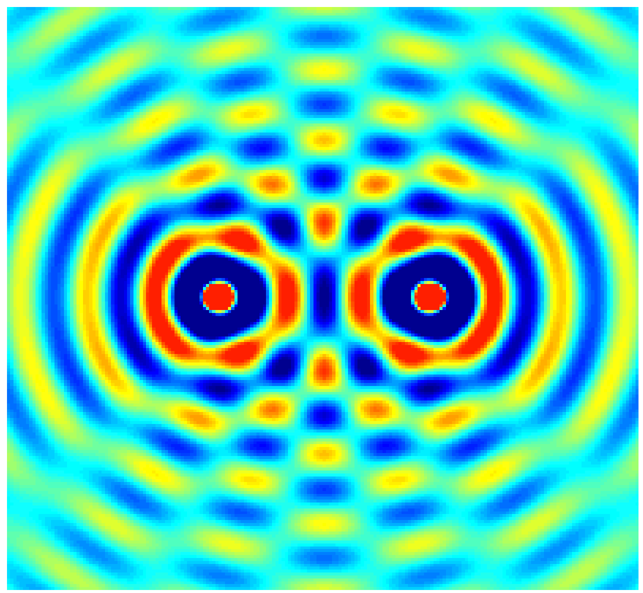
Applet Huygens Prin.

Interference

The term **interference** usually refers to the interaction of waves which are correlated or **coherent** with each other, either because they come from the same source or because they have the same or nearly the same frequency.



The interaction depend on **total phase difference** derived from the sum of both the path difference and the initial phase difference (if the waves are generated from 2 or more different sources). It can then be concluded whether the waves reaching a point are ***in phase*** (constructive interference) or ***out of phase*** (destructive interference).

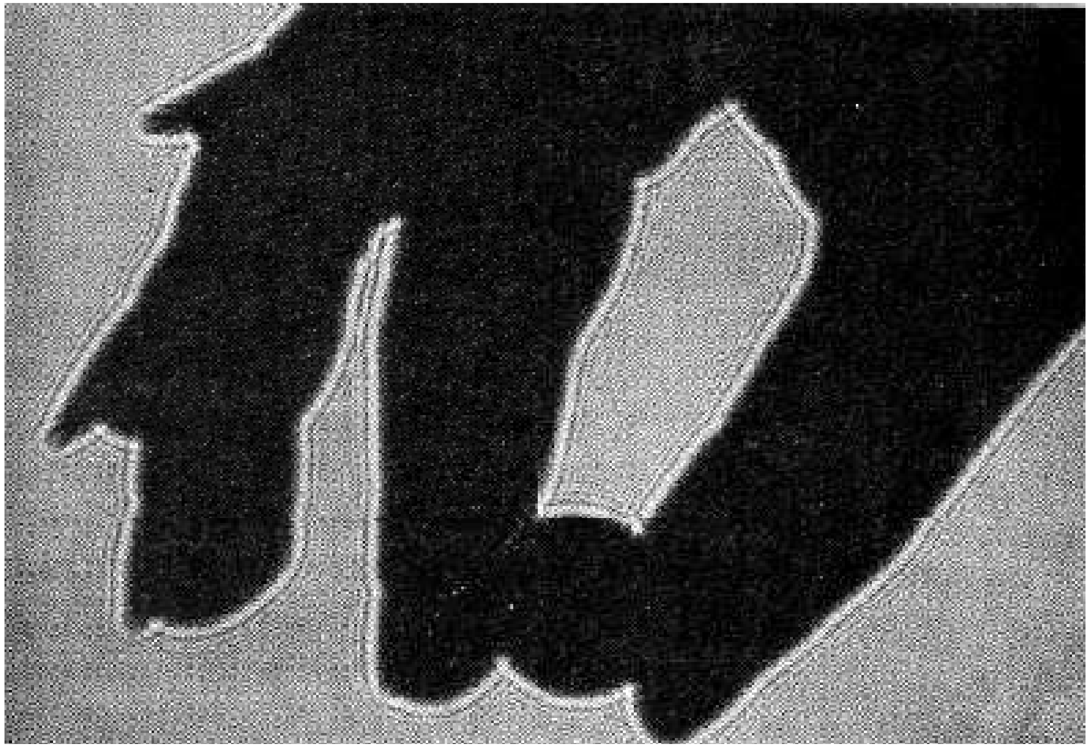


Diffraction

Light does not always travel in a straight line.

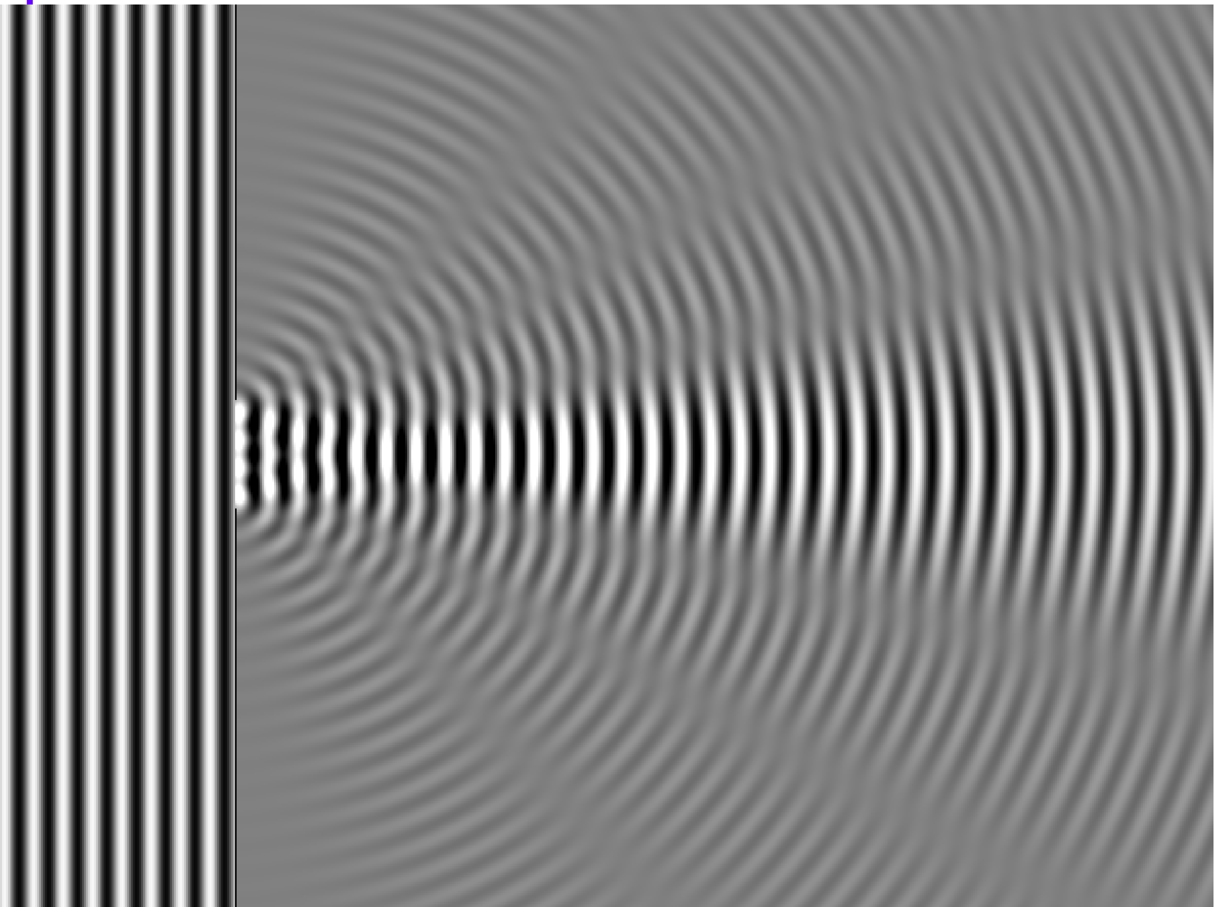
It tends to bend around objects. This tendency is called **diffraction**.

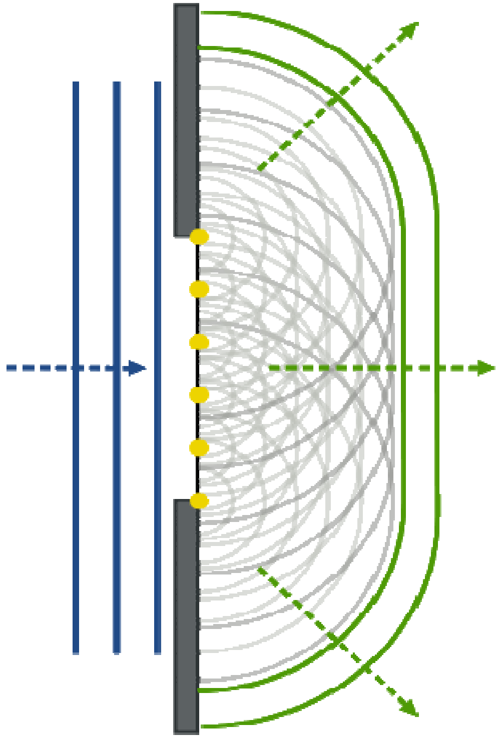
Any wave will do this, including matter waves and acoustic waves.



Shadow of a hand illuminated by a Helium-Neon laser

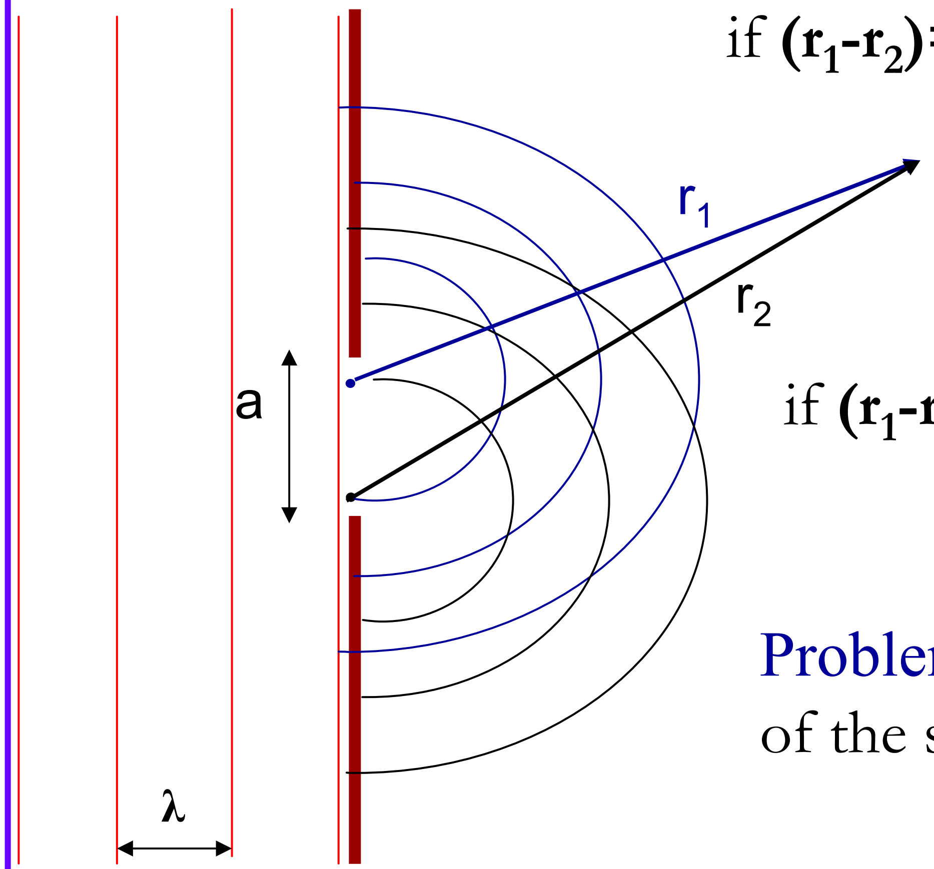
Diffraction and Huygens principle





Diffraction Applet

Slit Diffraction: the Idea



if $(\mathbf{r_1}-\mathbf{r_2})=\mathbf{m}\lambda \Rightarrow$ constructive interference

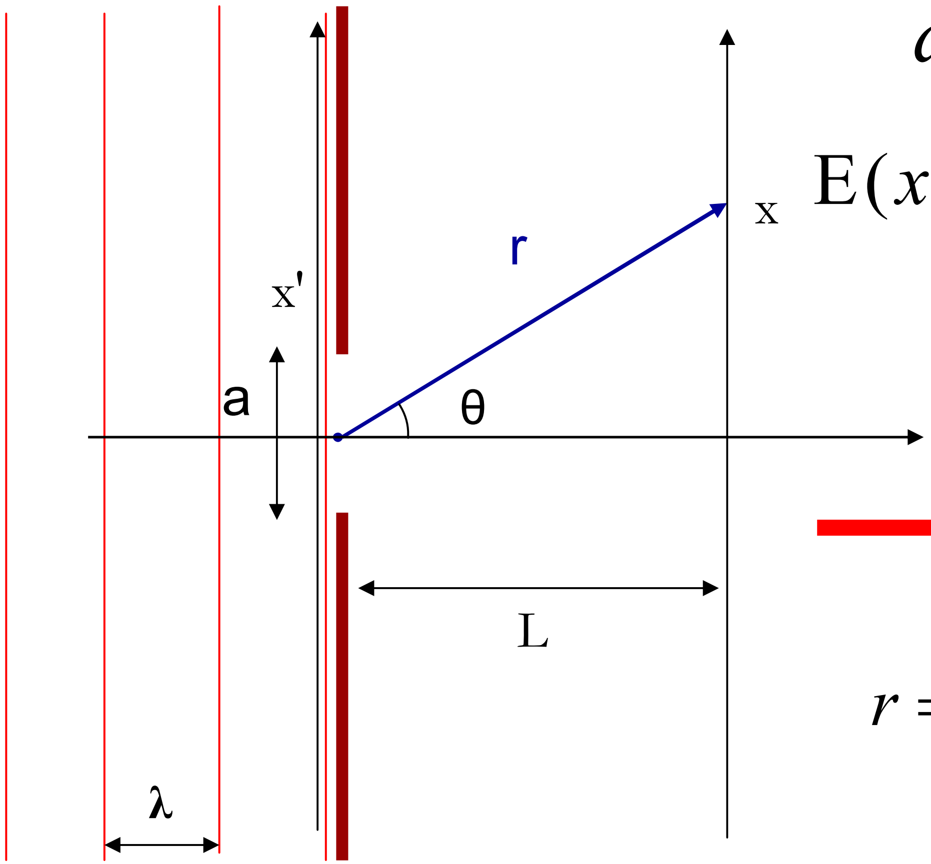
(**m** integer)

if $(\mathbf{r_1}-\mathbf{r_2})=\mathbf{n}\lambda/\mathbf{2} \Rightarrow$ destructive interference

(**n** integer odd)

Problem: we have to account for all the points of the slit !!!

Slit Diffraction: Starting



$$dE(x) = \frac{E(x')}{r} e^{i(kr-\omega t)} dx'$$

$$E(x') = cost = E_0 \quad \therefore \quad e^{i(kr-\omega t)} = e^{-i\omega t} e^{ikr}$$

$$\boxed{E(x) = E_0 e^{-i\omega t} \int_{-a/2}^{+a/2} \frac{e^{ikr}}{r} dx'}$$

$$r = \sqrt{(x-x')^2 + L^2} = L\sqrt{\frac{(x-x')^2}{L^2} + 1}$$

$$A = E_0 e^{-i\omega t}$$

$$\boxed{\sin\theta = \frac{x}{r}}$$

Slit Diffraction: Fraunhofer approx

L » | (x-x') | **far field**:

$$r \approx L\left(1+\frac{(x-x')^2}{2L^2}\right) \qquad \frac{1}{r} \approx \frac{1}{L} \qquad \boxed{\sin\theta = \frac{x}{r} \approx \frac{x}{L}}$$

$$e^{ikr} \approx e^{ikL\left(1+\frac{(x-x')^2}{2L^2}\right)} = e^{ikL} e^{ik\left(\frac{x^2-2xx'+x'^2}{2L}\right)} = e^{ikL} e^{\frac{ikx^2}{2L}} e^{-\frac{ikxx'}{L}} e^{\frac{ikx'^2}{2L}}$$

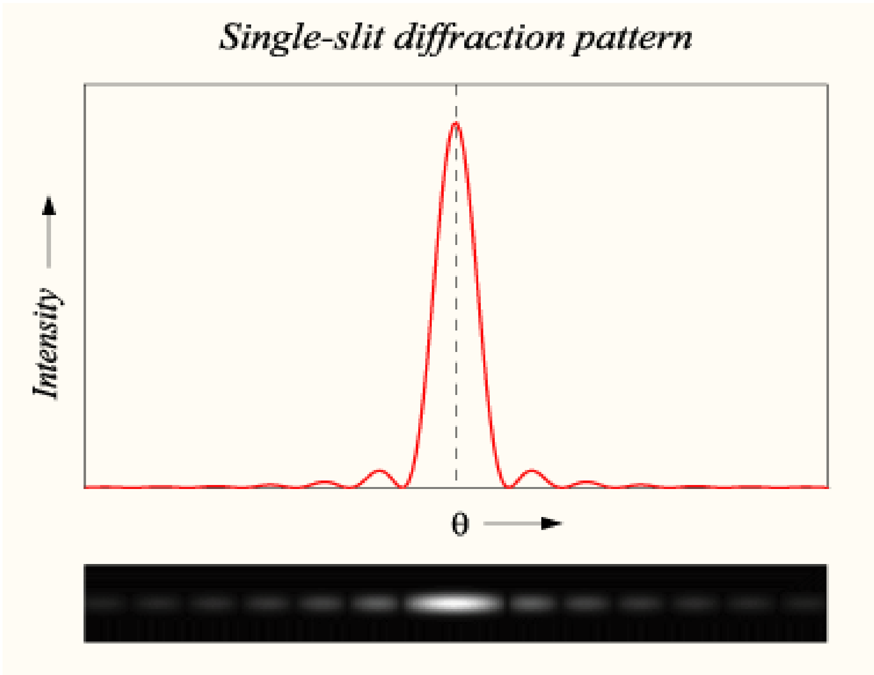
$$e^{\frac{ikx^2}{2L}} \approx 1 \;\therefore\; e^{\frac{ikx'^2}{2L}} \approx 1 \Rightarrow e^{ikr} \approx e^{ikL} e^{-\frac{ikxx'}{L}}$$

$$\boxed{\mathrm{E}(x) \approx \mathrm{E}_0 e^{-i\omega t} \frac{e^{ikL}}{L} \int_{-a/2}^{+a/2} e^{-\frac{ikxx'}{L}} dx' = C_1 \int_{-a/2}^{+a/2} e^{-\frac{ikxx'}{L}} dx'}$$

Slit Diffraction: the Solution

$$\mathrm{E}(x) \approx C_1 \frac{\left(e^{\frac{ikxa}{2L}} - e^{-\frac{ikxa}{2L}}\right)}{\frac{ikx}{L}} = aC_1 \frac{\sin \frac{ka\sin\theta}{2}}{\frac{ka\sin\theta}{2}}$$

$$\mathrm{I}(\theta) = I_0 \left(\frac{\sin \frac{ka\sin\theta}{2}}{\frac{ka\sin\theta}{2}} \right)^2$$



Slit Diffraction: Maxima (& minima)

Maxima:

$$\boxed{\frac{ka\sin\theta}{2} = \frac{\pi a\sin\theta}{\lambda} = (2m+1)\frac{\pi}{2} \Rightarrow \sin\theta = (2m+1)\frac{\lambda}{2a}}$$

m positive or negative integer; there is a $m_{\max}$?

Minima:

$$\boxed{\frac{ka\sin\theta}{2} = \frac{\pi a\sin\theta}{\lambda} = n\pi \Rightarrow \sin\theta = n\frac{\lambda}{a}}$$

n positive or negative integer; except *n*=0 where there is the main maximum

Slit Diffraction Applet

General behaviors:

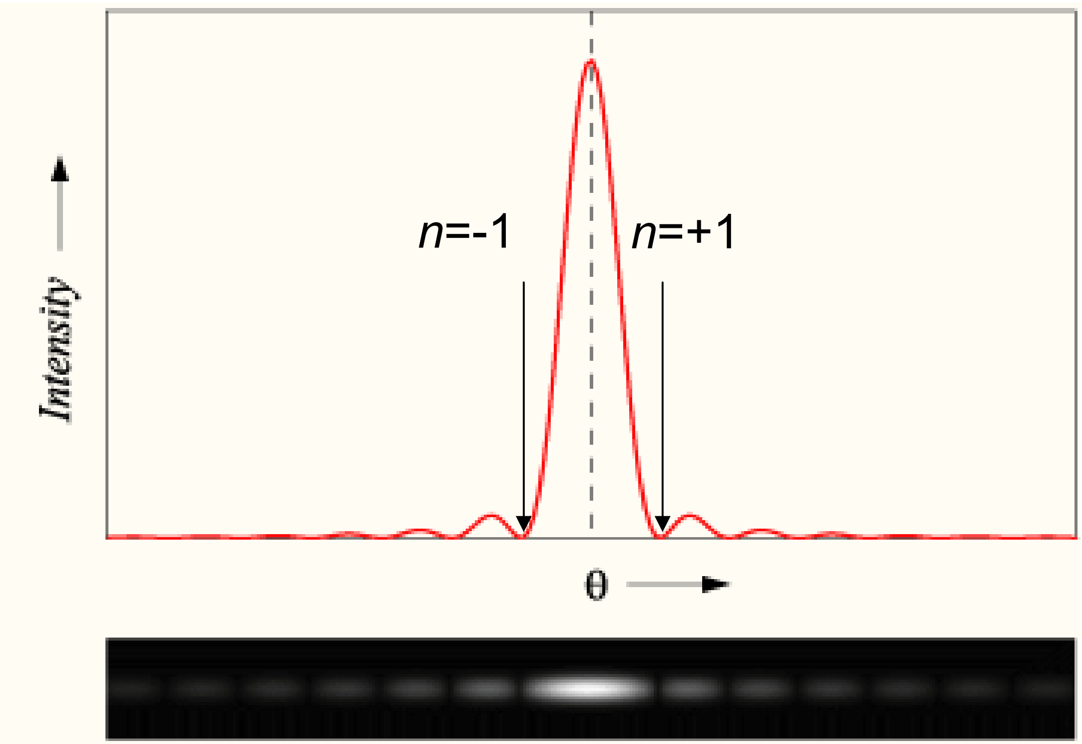
-smaller the width of diffracting slit 'wider' the resulting diffraction pattern;

-diffraction angles are invariant under scaling; that is, they depend only on the ratio of the wavelength to the size of the diffracting slit;

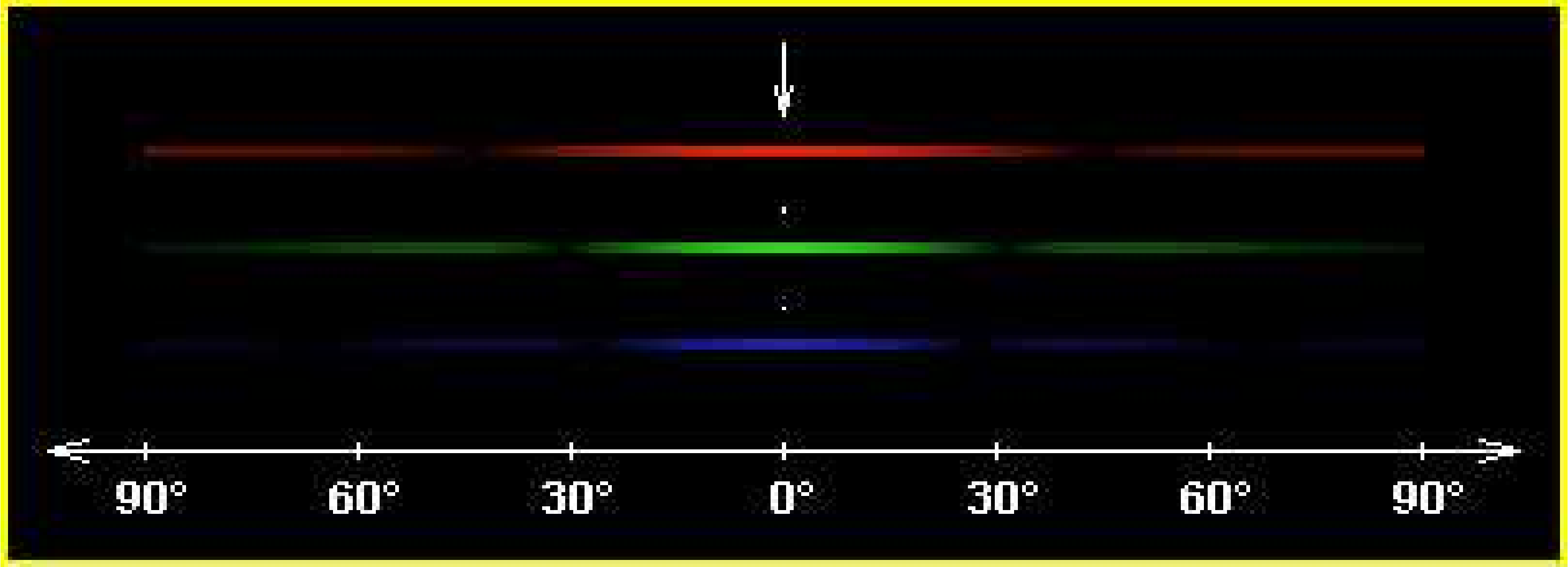
- different **λ's peak** at different **angles**.

Slit Diffraction Exercise

Determine the angular size of the diffraction pattern *"main lobe"* using a 5μm wide slit at λ= 629 nm (angular distance between *n*=-1 and *n*=+1 minima). [14.4^0]

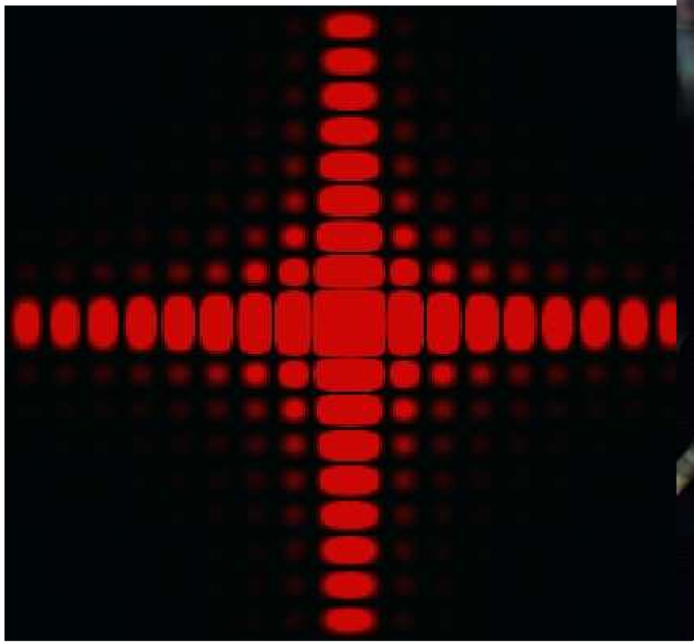


Slit Diffraction: Different λ



Different $\lambda \Rightarrow$ different location and wideness of maxima:
using *white light* $\Rightarrow$ no diffraction pattern but … blurred image

Nice Diffraction



Diffraction by squared slit

Diffraction by spider net