Waves and Optics - PHY204
(Smaldone - Sassi)

Gulu University

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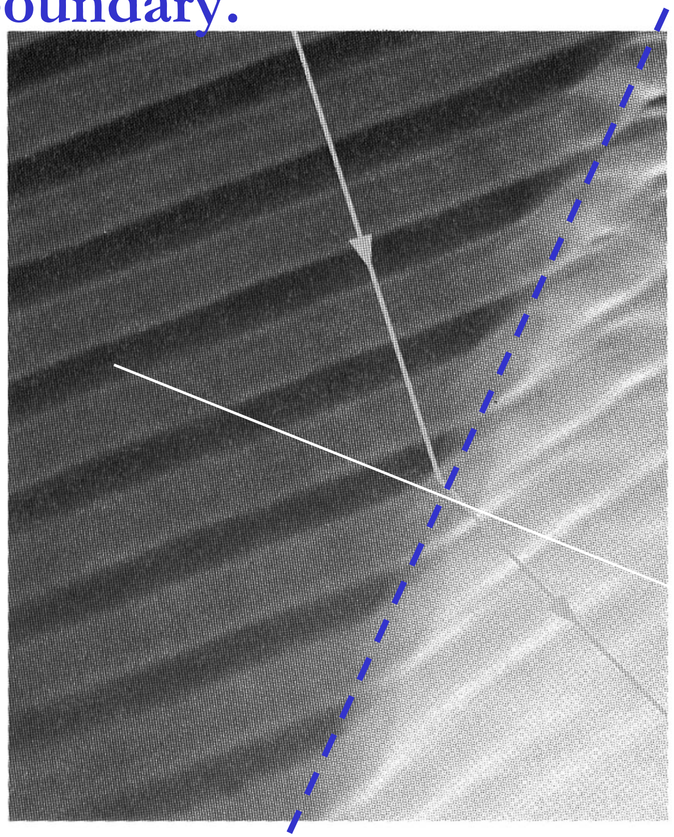
2

Refraction

Refraction - water waves…

Refraction – change in the direction of wave propagation at a boundary.

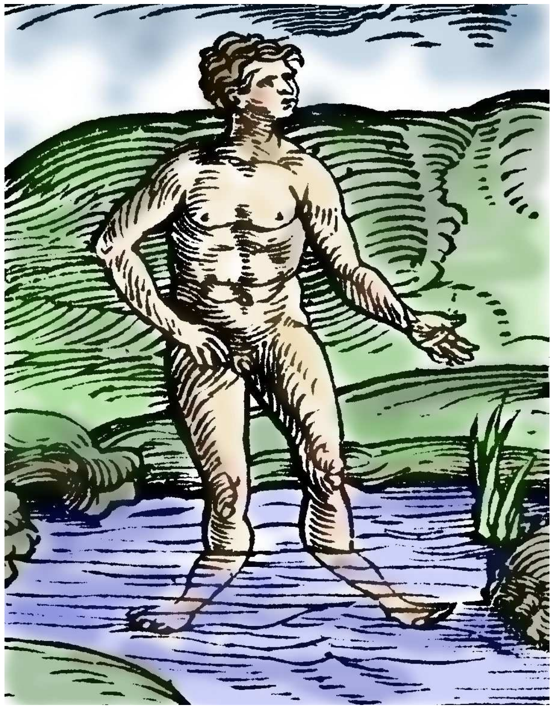


Waves, as they enter shallow water, slow down.

Waves in shallow water refract (change direction) at the interface between the different water depth

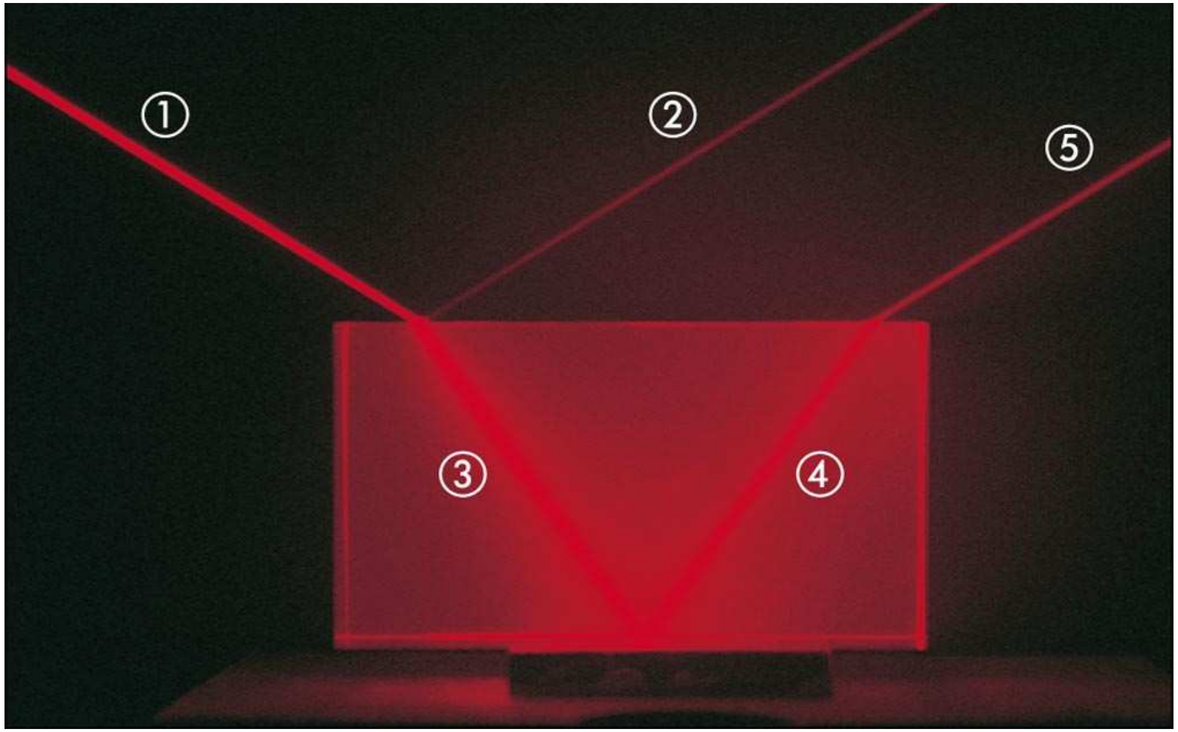
Refraction of Light - 1

- When a ray of light traveling through a transparent medium encounters a boundary leading into another medium, part of the ray is reflected and part of the ray enters the second medium
- The ray that enters the second medium is bent at the boundary
  - This bending of the ray is called ***refraction***



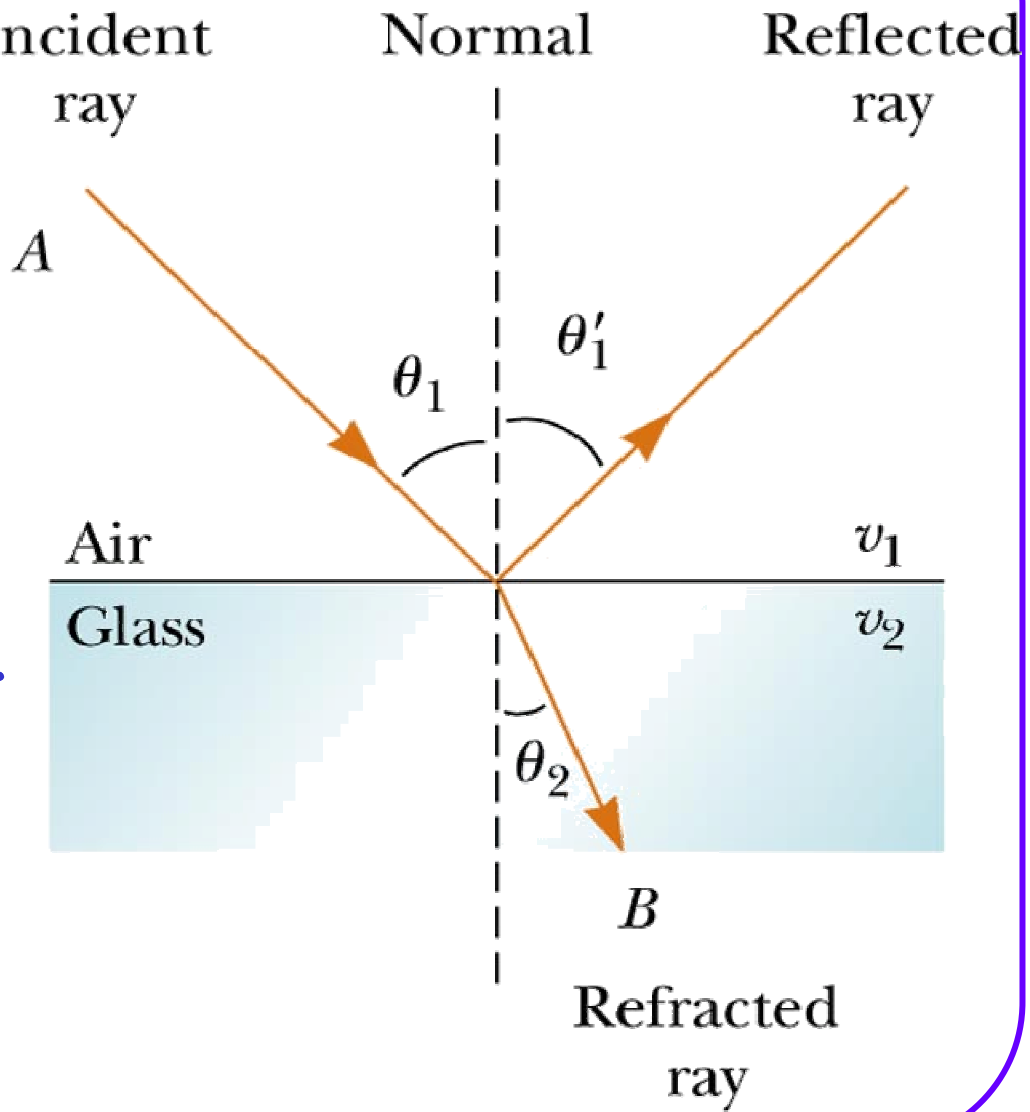
Following the Reflected and Refracted Rays

- Ray ① is the incident ray
- Ray ② is the reflected ray
- Ray ③ is refracted into the lucite
- Ray ④ is internally reflected in the lucite
- Ray ⑤ is refracted as it enters the air from the lucite



Refraction of Light - 2

- The incident ray, the reflected ray, the refracted ray, and the normal **all lie in the same plane**
- The angle of refraction, θ_2, depends on the **properties of the two media** (the speeds of light in them), $v_{1,2} = c/n_{1,2}$

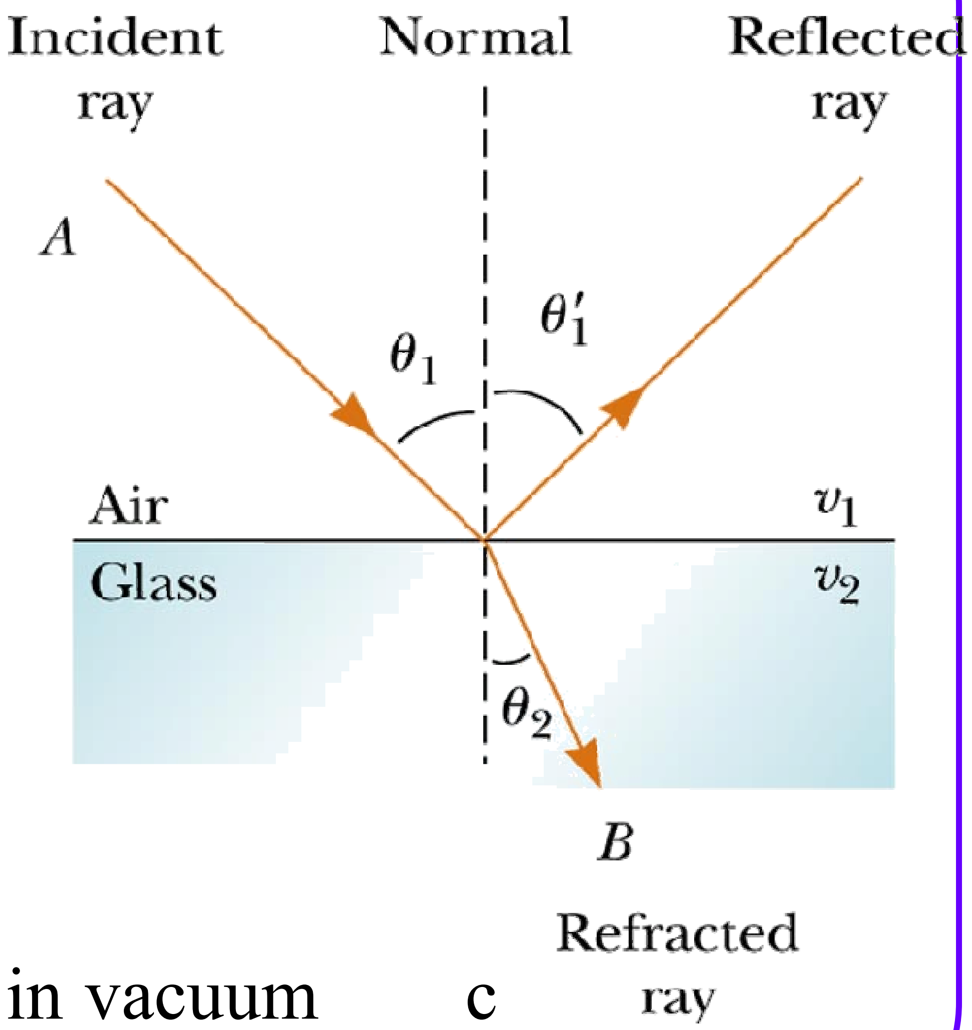


Refraction Law (Snell Law)

- The angle of refraction depends upon the two materials and the angle of incidence

$$\boxed{\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \text{constant}}$$

- The path of the light through the refracting surface is reversible
- Refraction occurs because the speeds of light, $\mathbf{v_1}$ and $\mathbf{v_2}$, are different in the two media
- The **index of refraction**, n, of a medium can be defined

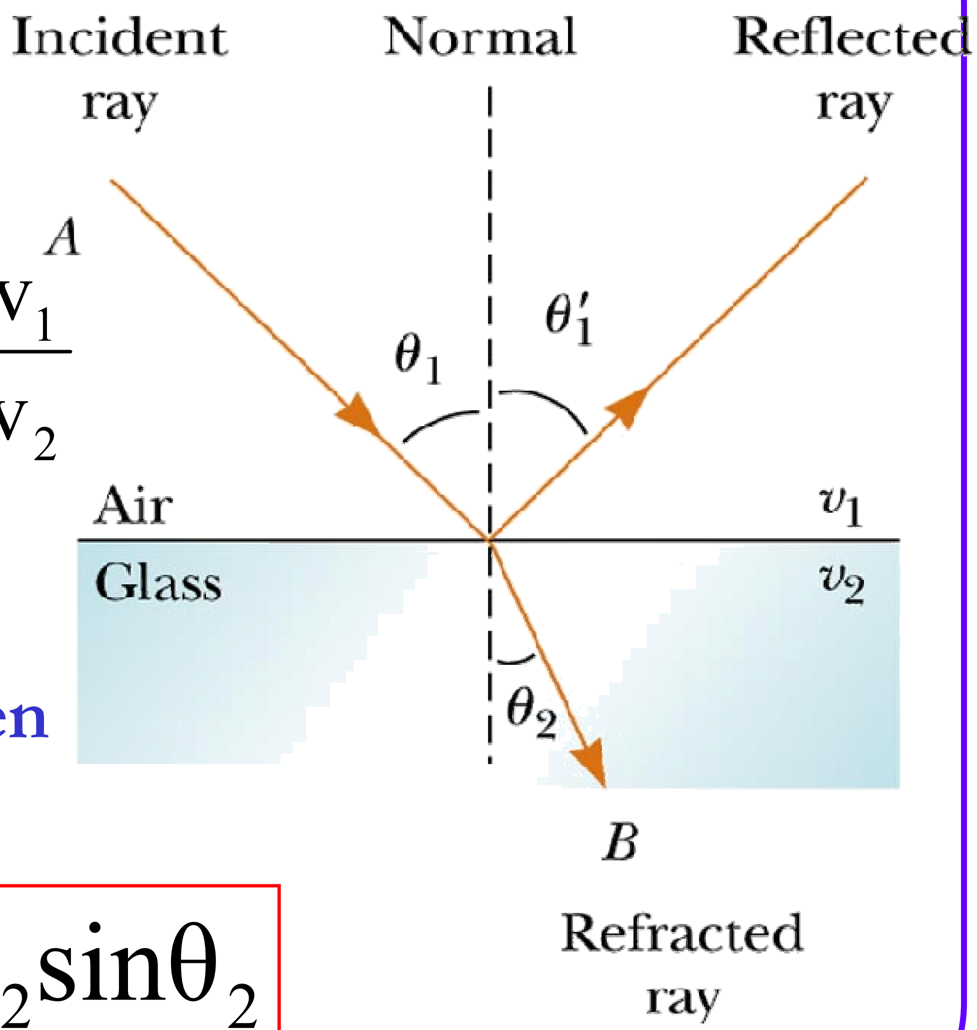


$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in the medium}} = \frac{c}{v}$$

Refraction Law and Speeds (of light)

- In terms of speeds the law of refraction becomes:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{c/v_2}{c/v_1} = \frac{v_1/c}{v_2/c} = \frac{v_1}{v_2}$$

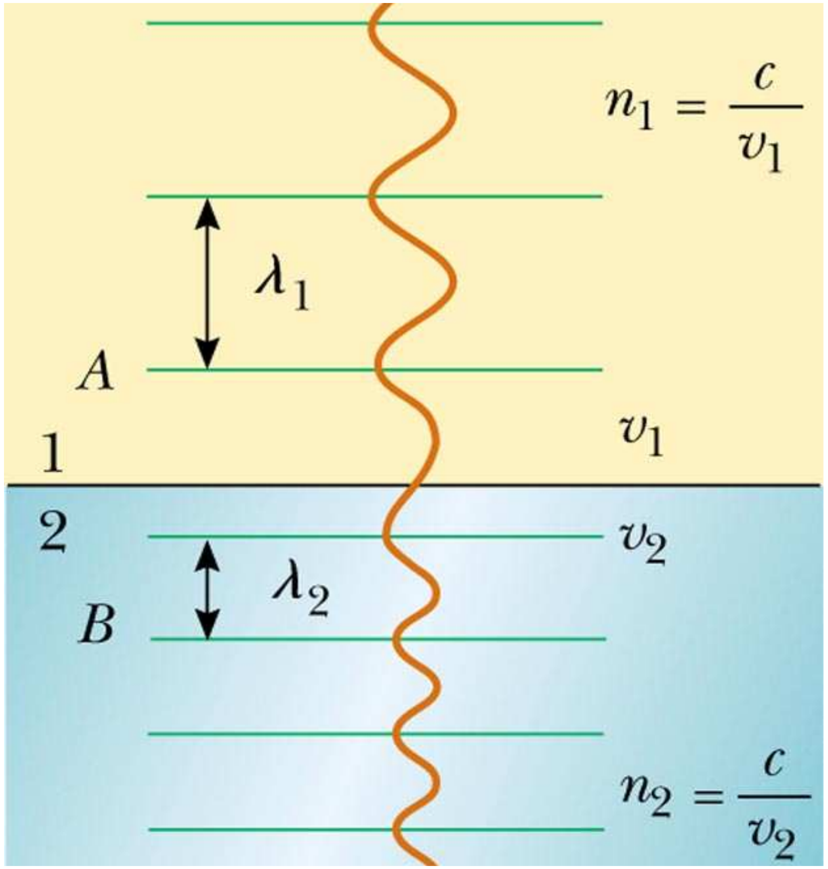


Snell's law of refraction is written in a form symmetric to the incident and refracted beams:

$$\boxed{n_1\sin\theta_1 = n_2\sin\theta_2}$$

Frequency and wavelength at refraction

- As light travels from one medium to another, *its frequency does not change*
  - Both the wave speed and the wavelength do change
  - The wavefronts do not pile up, nor are created or destroyed at the boundary, so f **must stay the same**



$$f_1 = f_2 \qquad \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

So we see that:

$$\lambda = n_1\lambda_1 = n_2\lambda_2$$

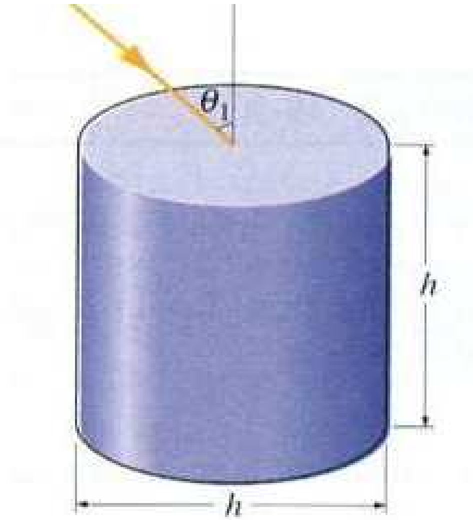
Table of Refraction Index

For a vacuum, n = 1

TABLE 22.1 **Indices of Refraction for Various Substances, Measured with Light of Vacuum Wavelength $\lambda_0 = 589$ nm**

Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		**Liquids at 20°C**	
Diamond (C)	2.419	Benzene	1.501
Fluorite (CaF_2)	1.434	Carbon disulfide	1.628
Fused quartz (SiO_2)	1.458	Carbon tetrachloride	1.461
Glass, crown	1.52	Ethyl alcohol	1.361
Glass, flint	1.66	Glycerine	1.473
Ice (H_2O) (at 0°C)	1.309	Water	1.333
Polystyrene	1.49		
Sodium chloride (NaCl)	1.544	**Gases at 0°C, 1 atm**	
Zircon	1.923	Air	1.000 293
		Carbon dioxide	1.000 45

Example of Snell's Law



What is the maximum θ_1 for which the beam will emerge through the bottom of the glass cylinder?

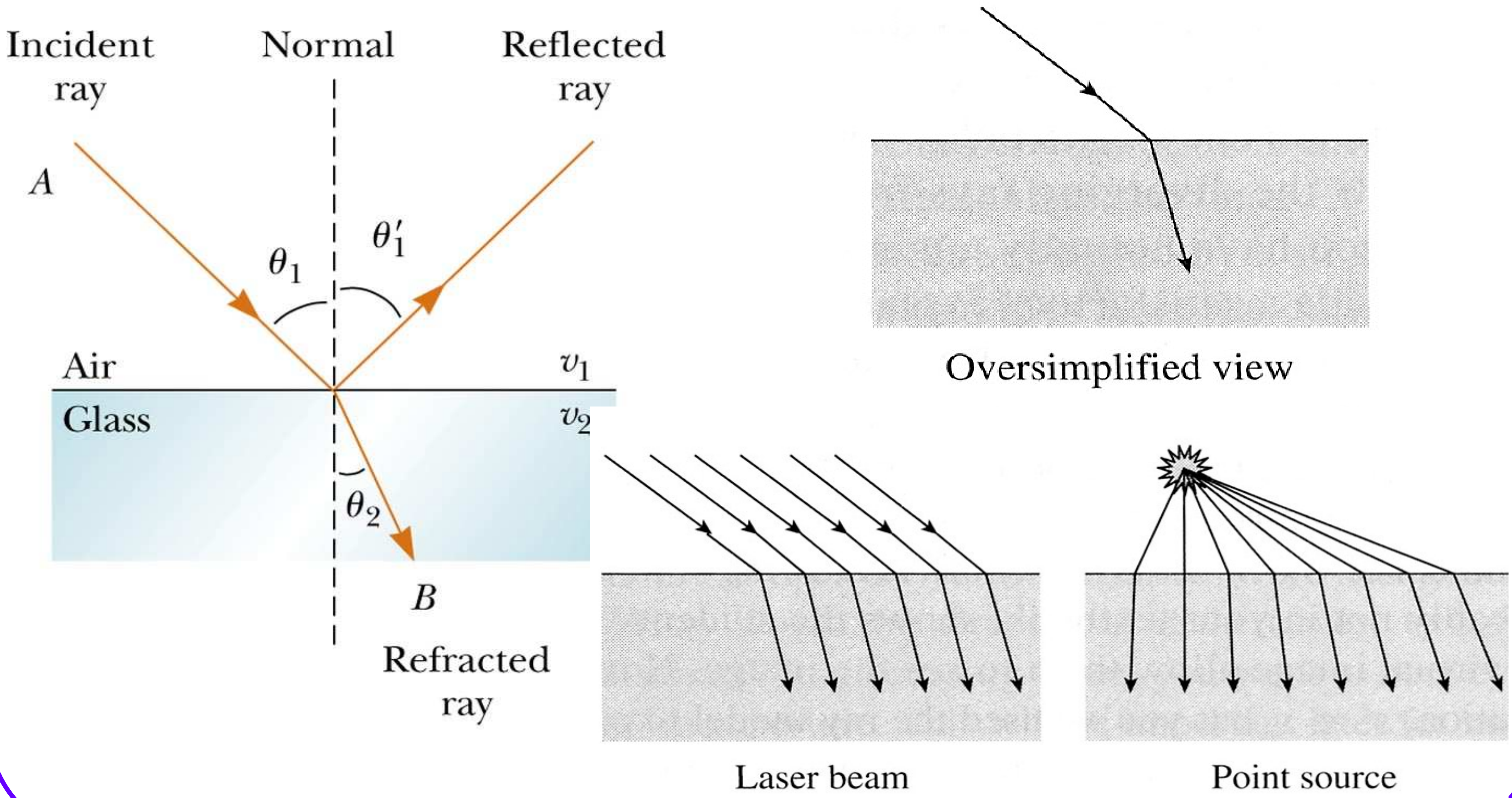
$$\left(n_1 \sin\theta_1 = n_2 \sin\theta_2, \ n_{air} = 1, \ n_{glass} = 1.52\right)$$

$$\sin\theta_2 = \frac{h/2}{\sqrt{(h/2)^2 + h^2}} = 1/\sqrt{5} = 0.447$$

$$\sin\theta_1 = n_{glass} \sin\theta_2 = 1.52 \times 0.447 = 0.68$$

$$\theta_1 = \sin^{-1} 0.68 = 42.80^\circ$$

Refraction of Light - 3



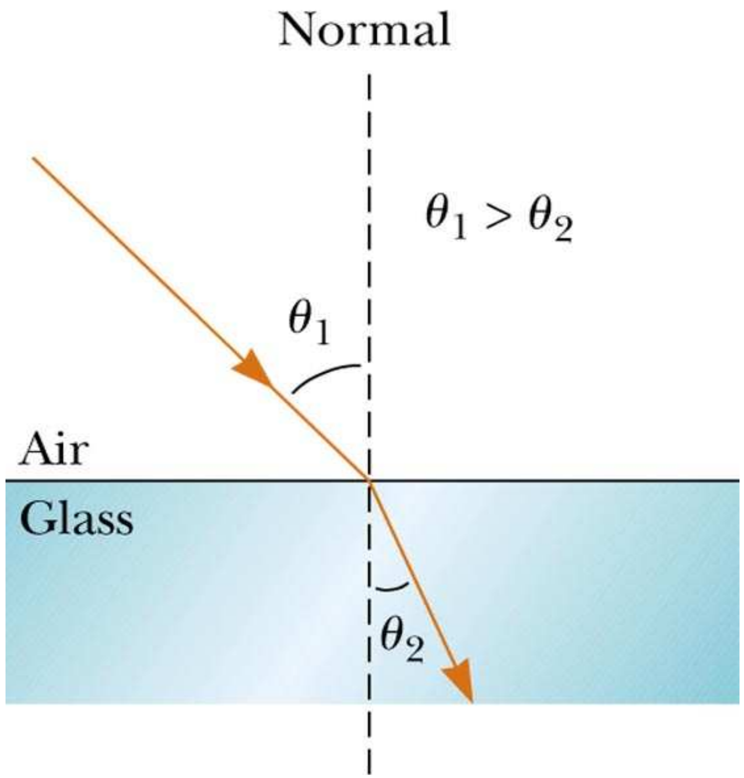
Refraction of Light - 4

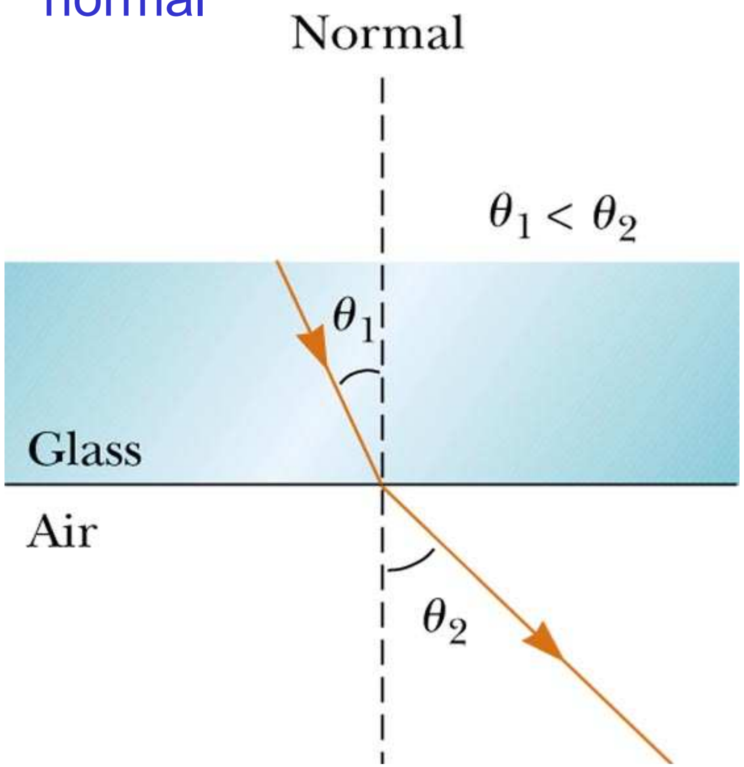
When light refracts into a material, where the **index of refraction is higher**, the angle of refraction is **less** than the angle of incidence

The ray **bends toward** the normal

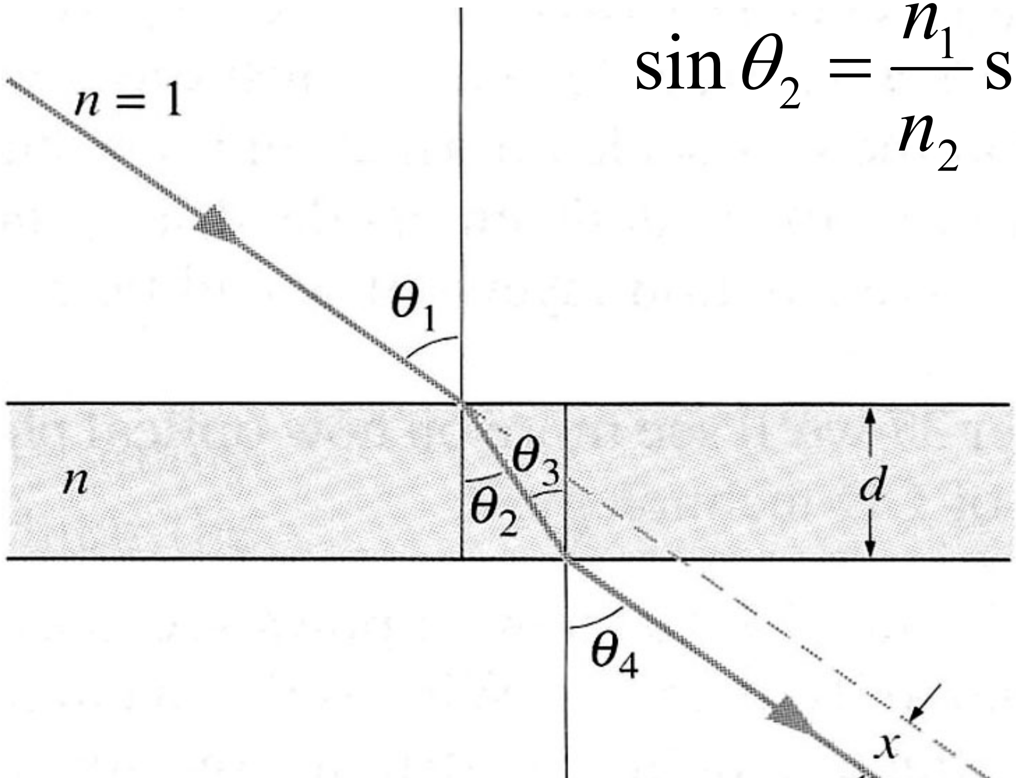
When light refracts into a material, where the **index of refraction is lower**, the angle of refraction is **greater** than the angle of incidence

The ray **bends away from** the normal





A light beam through a slab of glass:



$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

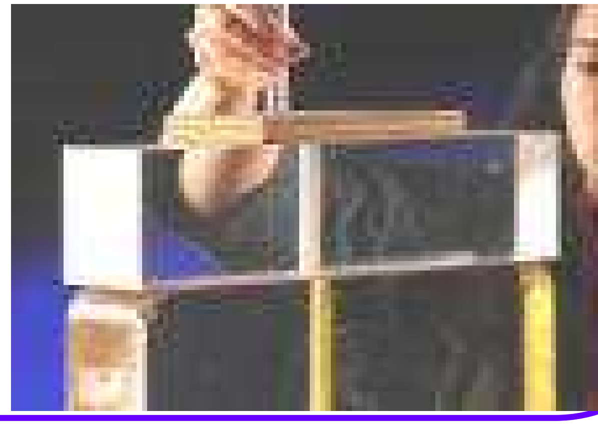
$$\sin\theta_4 = \frac{n_2}{n_1}\sin\theta_3$$

$$\theta_3 = \theta_2$$

$$\sin\theta_4 = \frac{n_2}{n_1}\frac{n_1}{n_2}\sin\theta_1$$

$$\boxed{\theta_4 = \theta_1}$$

A light beam incident on a transparent slab emerges with its original direction, but is displaced from its original path



Light and Diamond

Diamond has a refractive index of n=2.42. For light with a wavelength of λ= 589nm find the (a) frequency, (b) light speed, and (c) wavelength in diamond.

The frequency is continuous across the boundary!

(a) $f' = f,\ f' = c/\lambda = 3 \times 10^8/589 \times 10^{-9} = 5.1 \times 10^{14} Hz$

(b) $v = c/n = (3 \times 10^8 m/\sec)/2.42 = 1.24 \times 10^8 m/\sec$

(c) $\lambda' = v/f = c/fn = \lambda/n = 589nm/2.42 = 243nm$

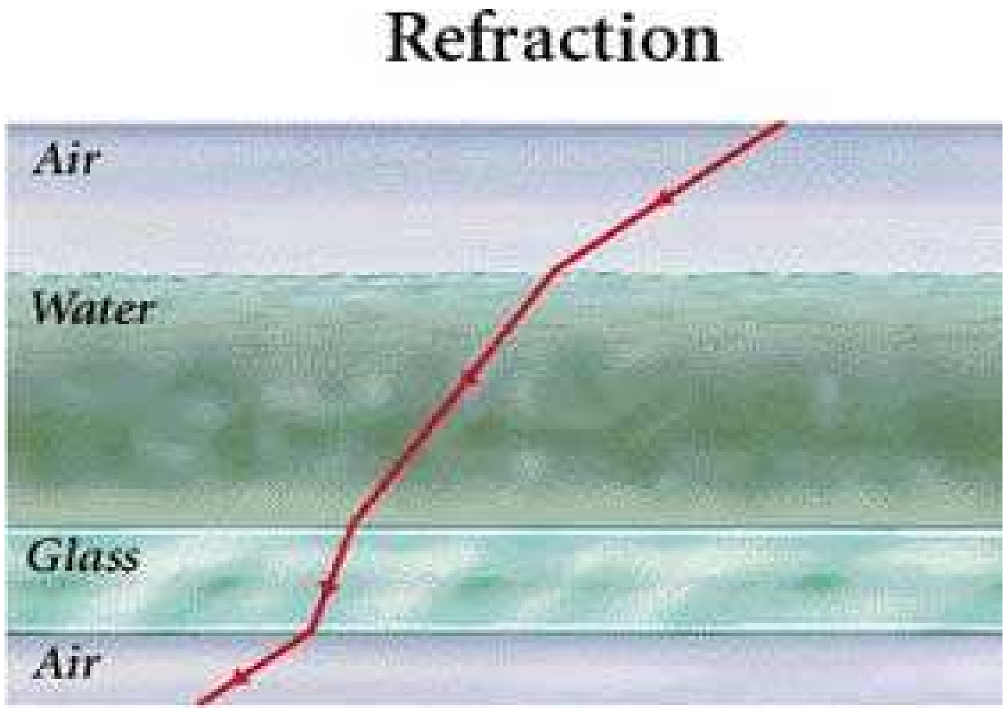
Color of the Sun from under water.

- The Sun looks yellow, since its radiation intensity has a maximum at $\lambda = 550\ nm$, which is yellow light.
- Wavelength of this yellow light in water will be $\lambda' = \lambda / n_{water} = \lambda/1.33 = 413\ nm$, which corresponds to violet light.
- Is the Sun going to look violet from under water?

- Of course not! The only thing that matters is the wavelength inside your eye, which is defined by n of your vitreous humor.

$$f_{eye} = f_{air} \qquad \lambda_{eye} = \lambda_{air} \frac{n_{air}}{n_{eye}}$$

Example of Refraction Law



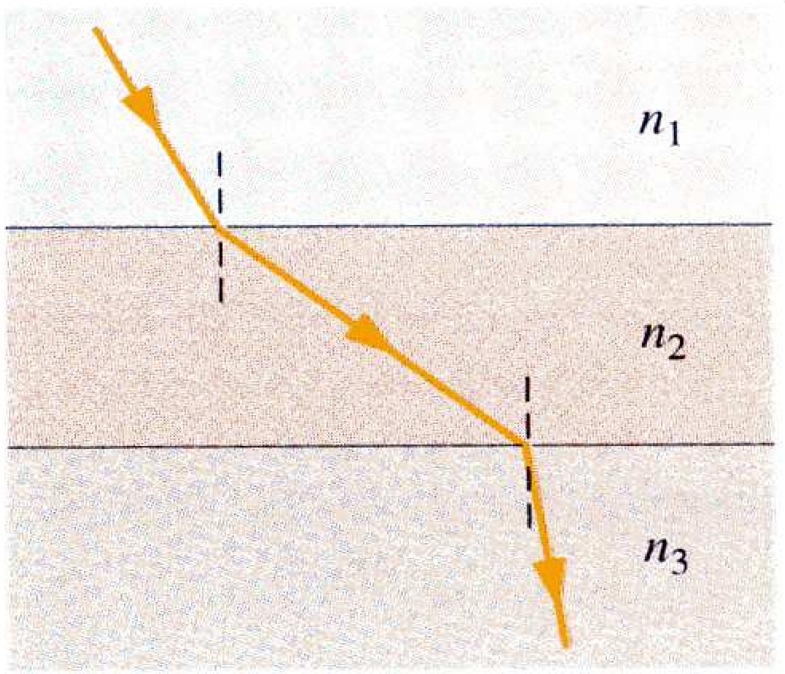
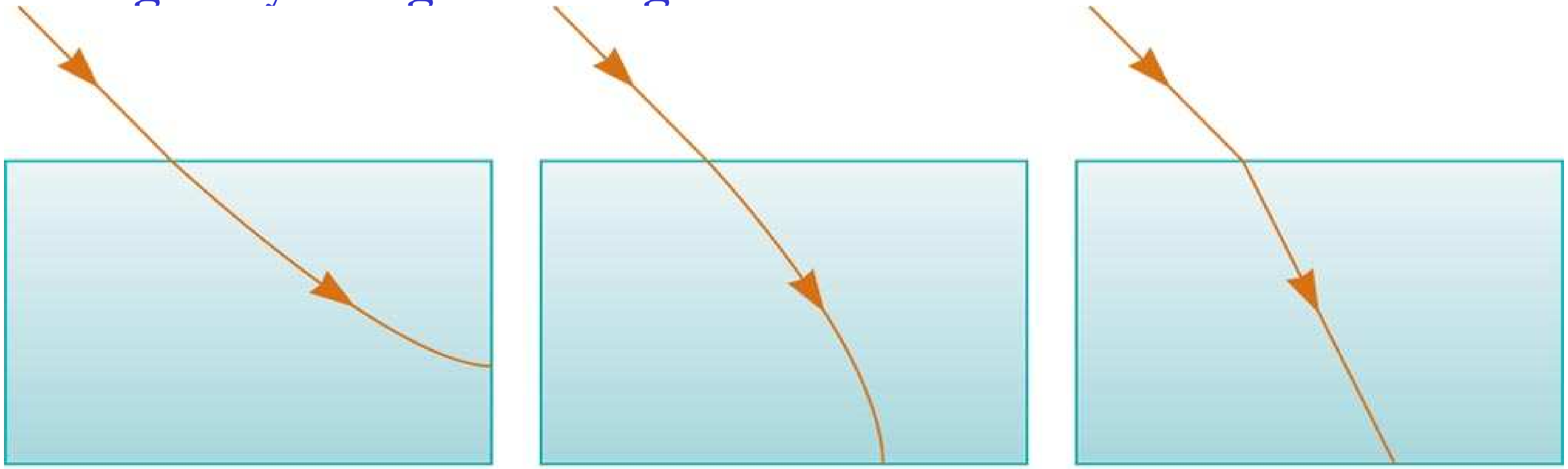


FIGURE 35-11 How do the refractive indices compare?

$$n_3 > n_1 > n_2$$

Another Example of Refraction Law

A material has an index of refraction that increases continuously from top to bottom. Of the three paths shown in the figure below, which path will be the path of a light ray as it goes through the material?



(a) (b) (c)

(b). When light goes from one material into one having a higher index of refraction, it refracts toward the normal line of the boundary between the two materials. If, as the light travels through the new material, the index of refraction continues to increase, the light ray will refract more and more toward the normal line.

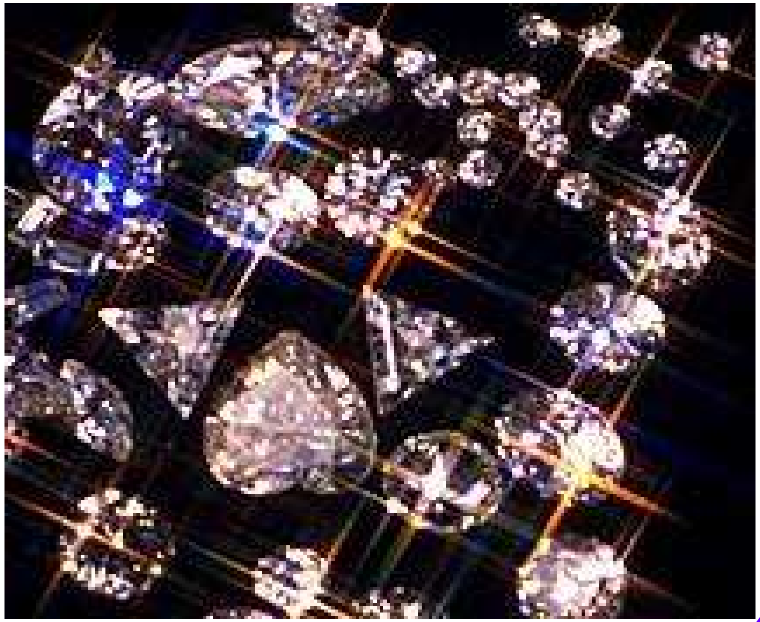
Reflection and refraction: diamonds

- **For a light beam incident upon a boundary between two transparent media at 90°:**

$$\frac{I_r}{I_i} = \frac{\text{Intensity of reflected beam}}{\text{Intensity of incident beam}} = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

- Glass – $n_2 = 1.5$, $\frac{I_r}{I_i} = 0.04$

- Diamond – $n_2 = 2.4$, $\frac{I_r}{I_i} = 0.17$



What happens if the light is not coming at the angle of 90°?

Intensity of Reflected Beam

What happens if the light is not coming at the angle of 90°?

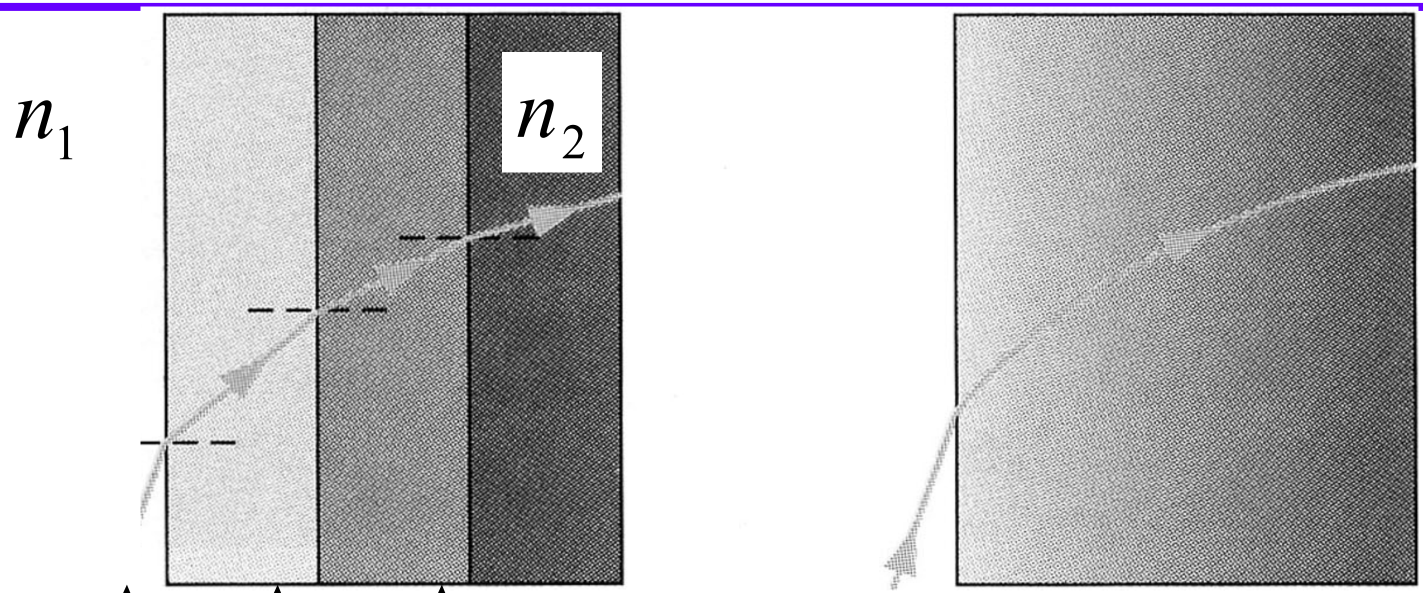
The light coming at 90° corresponds to the angle of incidence $\theta = 0$.

As a rule, at greater angles of incidence more light gets reflected.

Look at your watch glass at a grazing angle! It will act as a pretty good mirror!

Is there any dependence on polarization of light?

Antireflection coating



$\frac{\Delta n}{3}$ $\frac{\Delta n}{3}$ $\frac{\Delta n}{3}$ $\Delta n = n_2 - n_1$

Total intensity of light reflected from all 3 interfaces

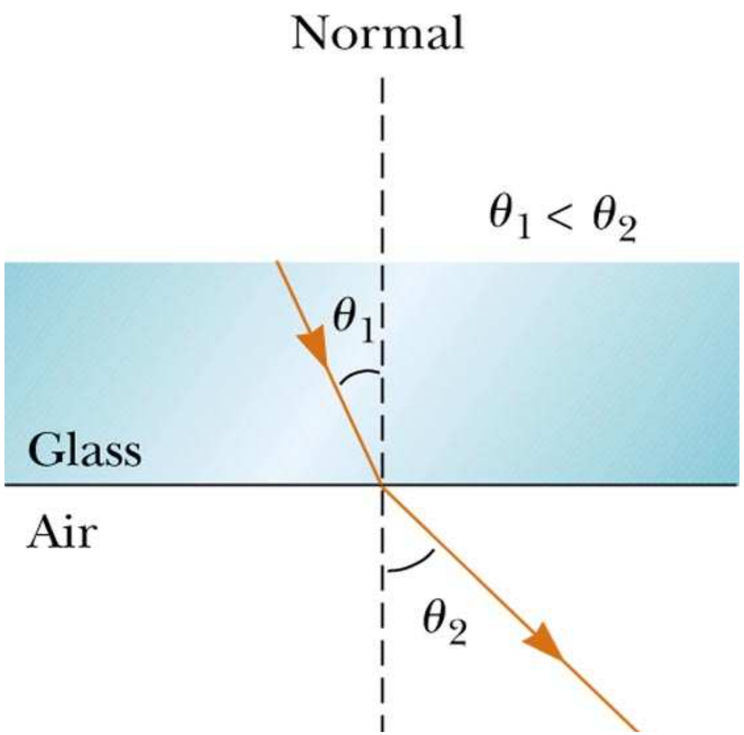
$$I_r = 3 \cdot I_i \frac{(\Delta \mathrm{n}/3)^2}{(\mathrm{n}_1 + \mathrm{n}_2)^2} = \frac{1}{3} \cdot I_i \frac{(\Delta \mathrm{n})^2}{(\mathrm{n}_1 + \mathrm{n}_2)^2}$$

By using multiple interfaces we can significantly reduce reflections – antireflection coatings.

Strange Refraction

When light refracts into a material, where the **index of refraction is lower**, the angle of refraction is **greater** than the angle of incidence

The ray **bends away from the normal**



Snell's law of refraction :

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

We have $n_1 > n_2$

What if θ_1 is so large that also

$$n_1 \sin\theta_1 > n_2$$

That we always have

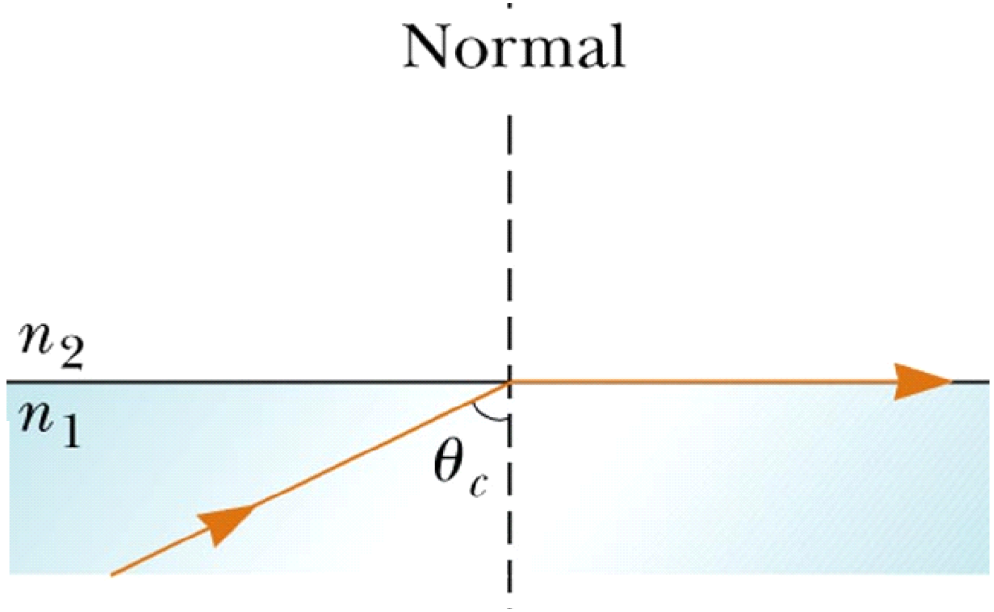
$$n_1 \sin\theta_1 > n_2 \sin\theta_2$$

?

Critical Angle

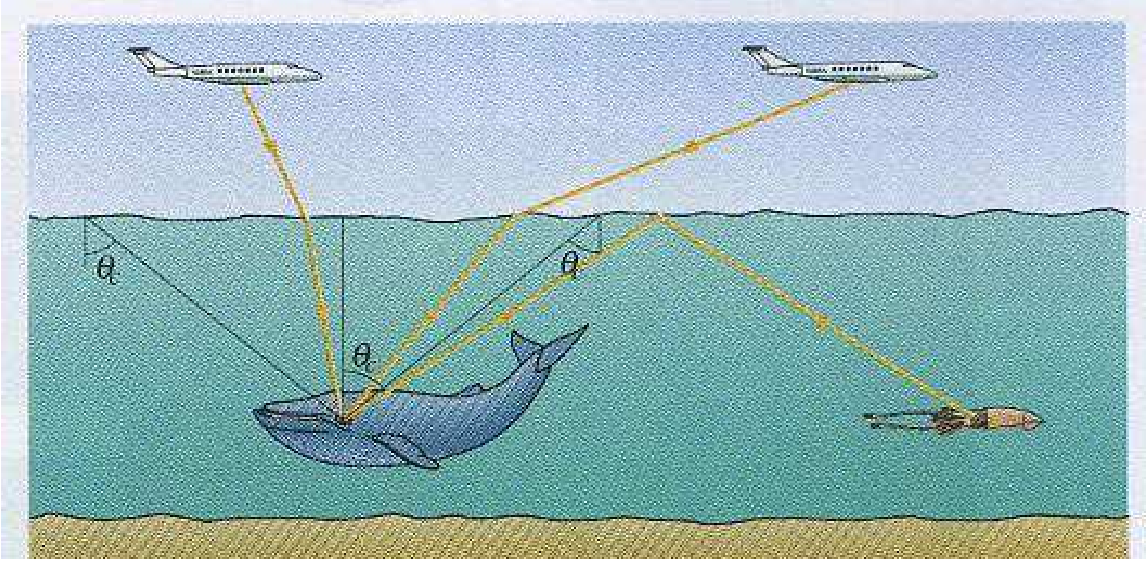
- A particular angle of incidence will result in an angle of refraction of 90°
  - This angle of incidence is called the *critical angle*

$$\sin\theta_c = \frac{n_2}{n_1},\ n_1 > n_2$$



- For angles of incidence *greater* than the critical angle, the beam is entirely reflected at the boundary
  - This ray obeys the Law of Reflection at the boundary
- Total internal reflection occurs only when light attempts to move from a medium of higher index of refraction to a medium of lower index of refraction

Fish Watch



$$\sin\theta_c = \frac{n_2}{n_1},\ n_1 > n_2$$

The fish sees the entire world above surface in a cone of half angle θ_c. Looking beyond this cone, it sees … ??

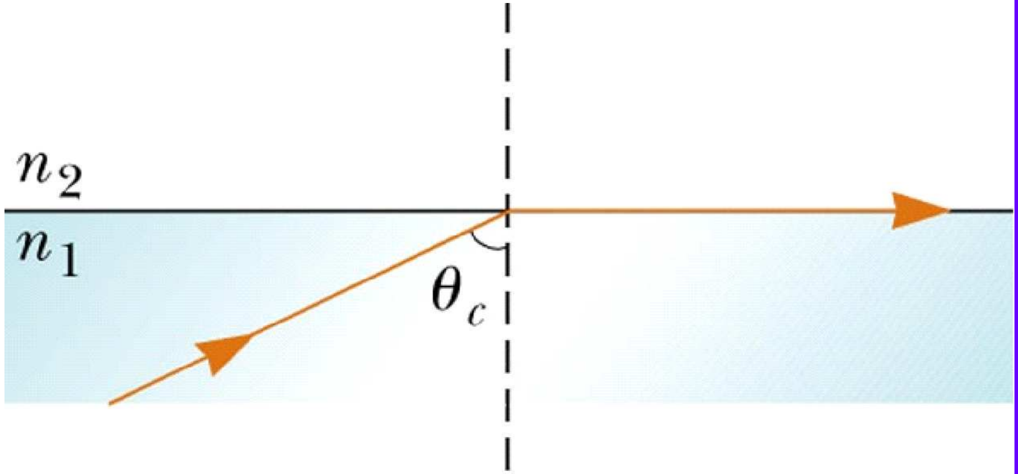
$$\sin\theta_c = 1/1.333 = .75 \rightarrow \theta_c = .848 rad = 48.6^\circ$$

And what does the fish see beyond the cone??

Reflections back into the water, other creatures of the deep.

Critical Angles

$$\sin\theta_c = \frac{n_2}{n_1} \quad \text{for } n_1 > n_2$$



Glass and air: $$\frac{n_2}{n_1} = \frac{1}{1.5} \qquad \theta_c = \sin^{-1}(2/3) = 42°$$

Water and air: $$\frac{n_2}{n_1} = \frac{1}{1.33} \qquad \theta_c = \sin^{-1}(0.75) = 49°$$

Air and vacuum: $$\frac{n_2}{n_1} = \frac{1}{1.00022} \qquad \theta_c = 88.8°$$

Hot and Cold Air

- Critical angle - an angle of incidence which result in an angle of refraction of 90°

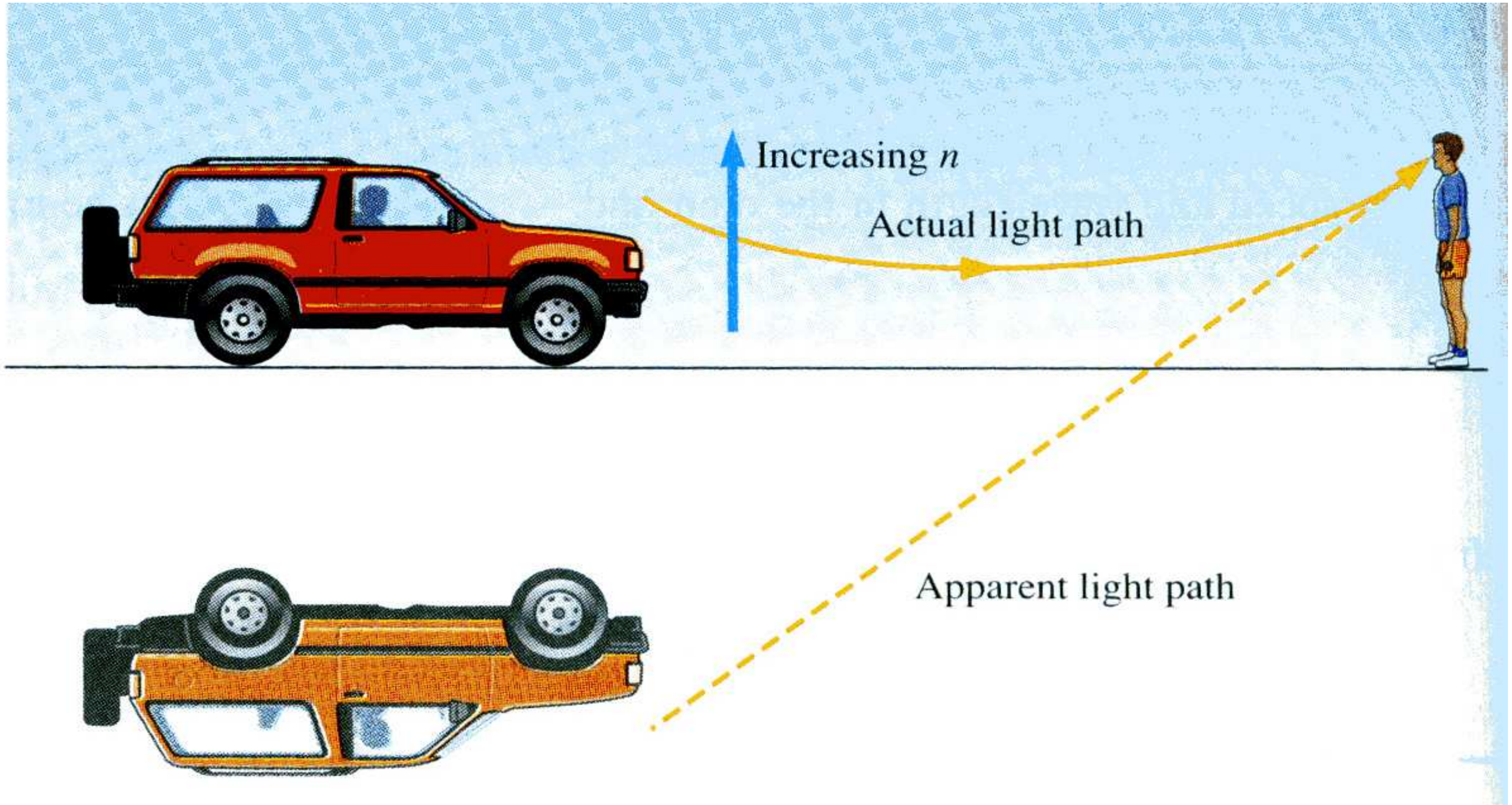
Cold air and hot air with 10% lower index of refraction

$$\frac{n_2}{n_1} = \frac{1.0002}{1.00022}$$

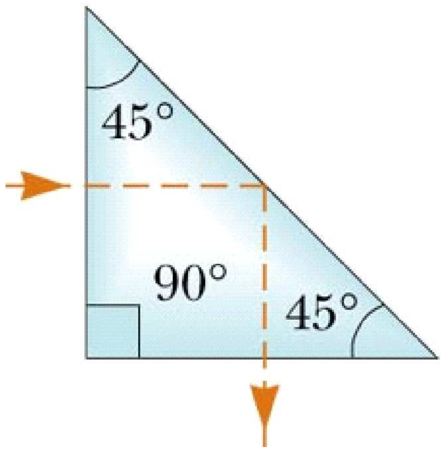
$$\theta_c = 89.6°$$

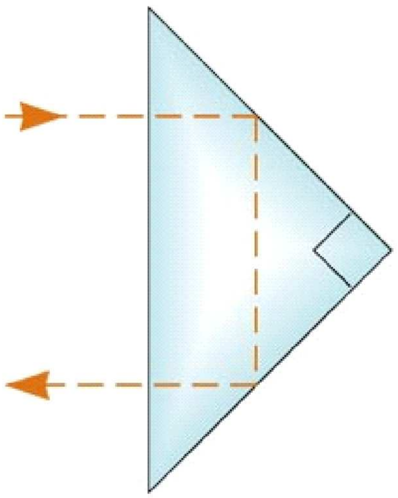
It is still 0.4° from the surface!

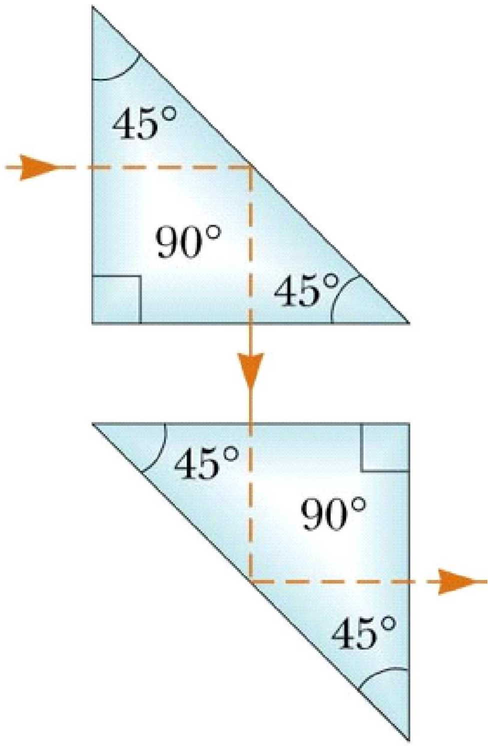
Optical Illusions

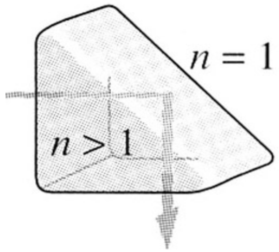


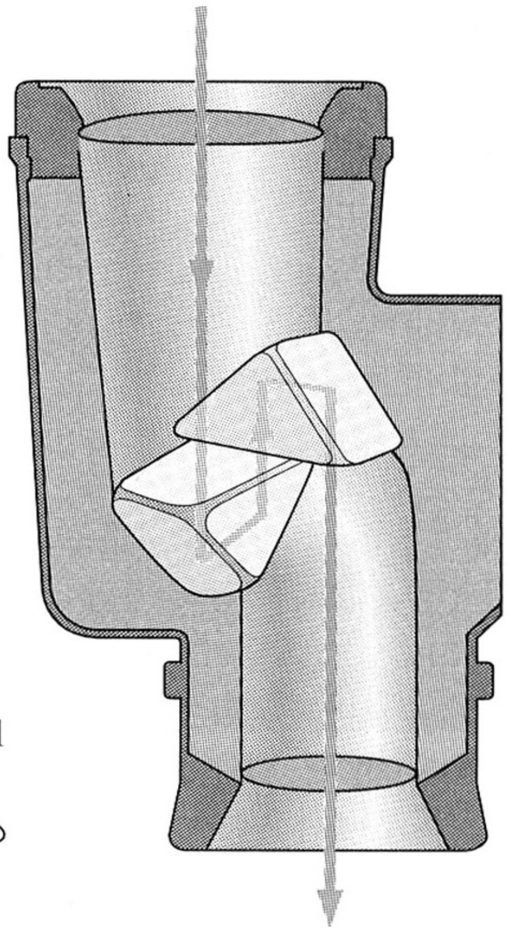
Prisms as excellent reflectors











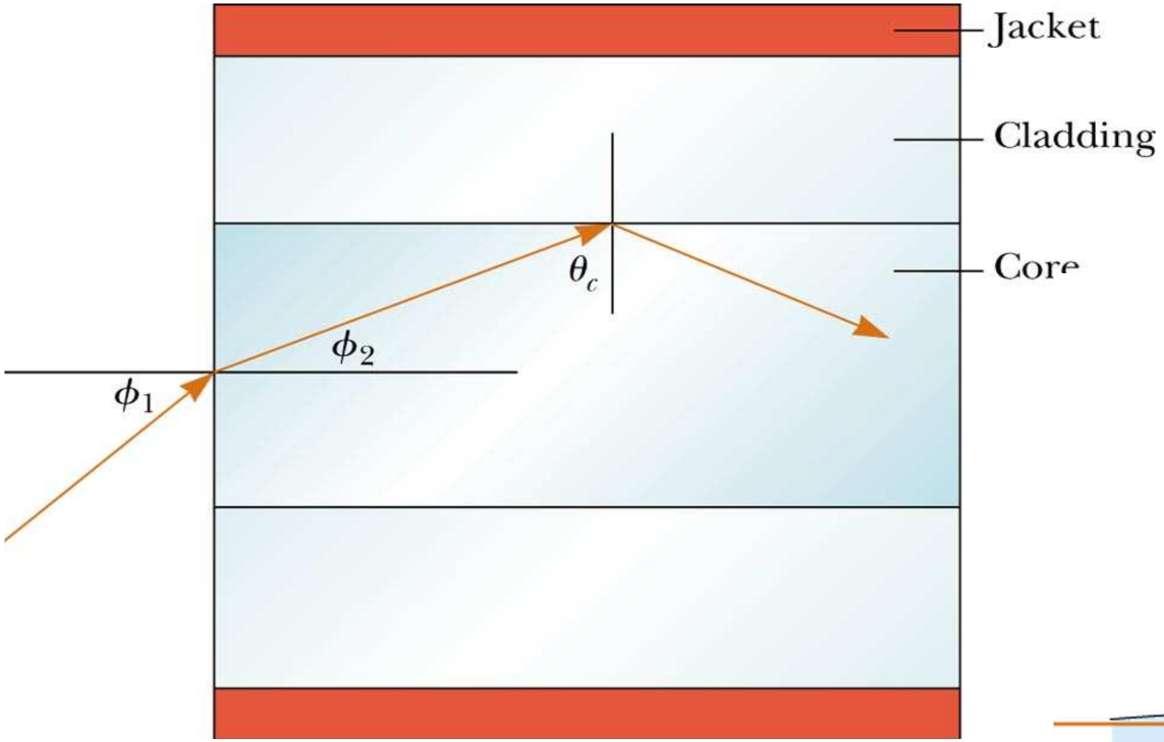
Optical Fibers

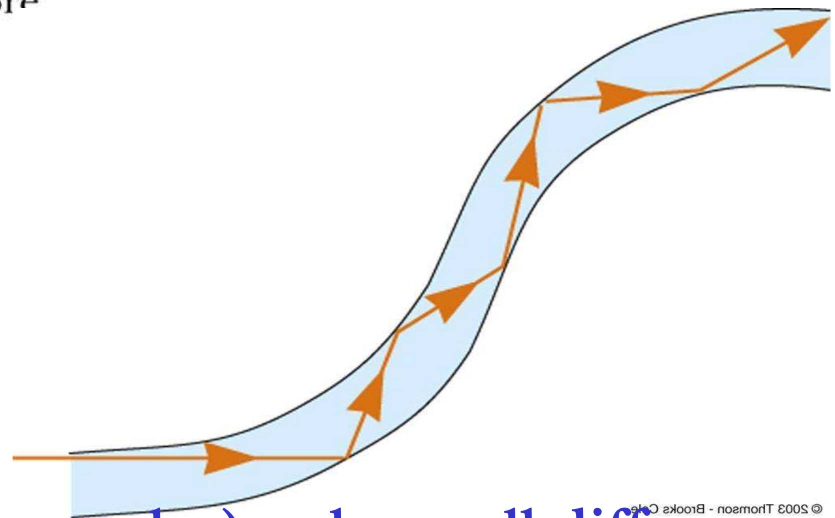
Total internal reflection at the boundaries between the core and cladding.

$$\sin\theta_c > \frac{n_{cladding}}{n_{core}}$$

$$\theta_c = 90° - \phi_2$$

$$\sin\phi_2 = \sin\phi_1 / n_{core}$$

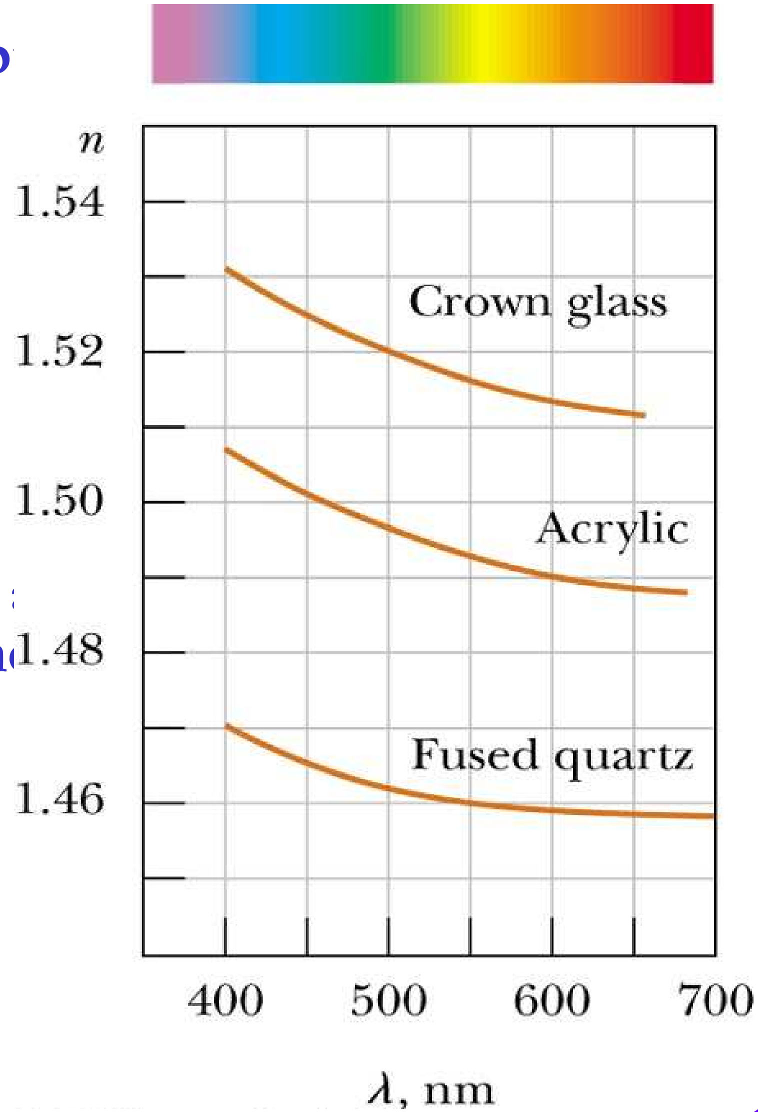




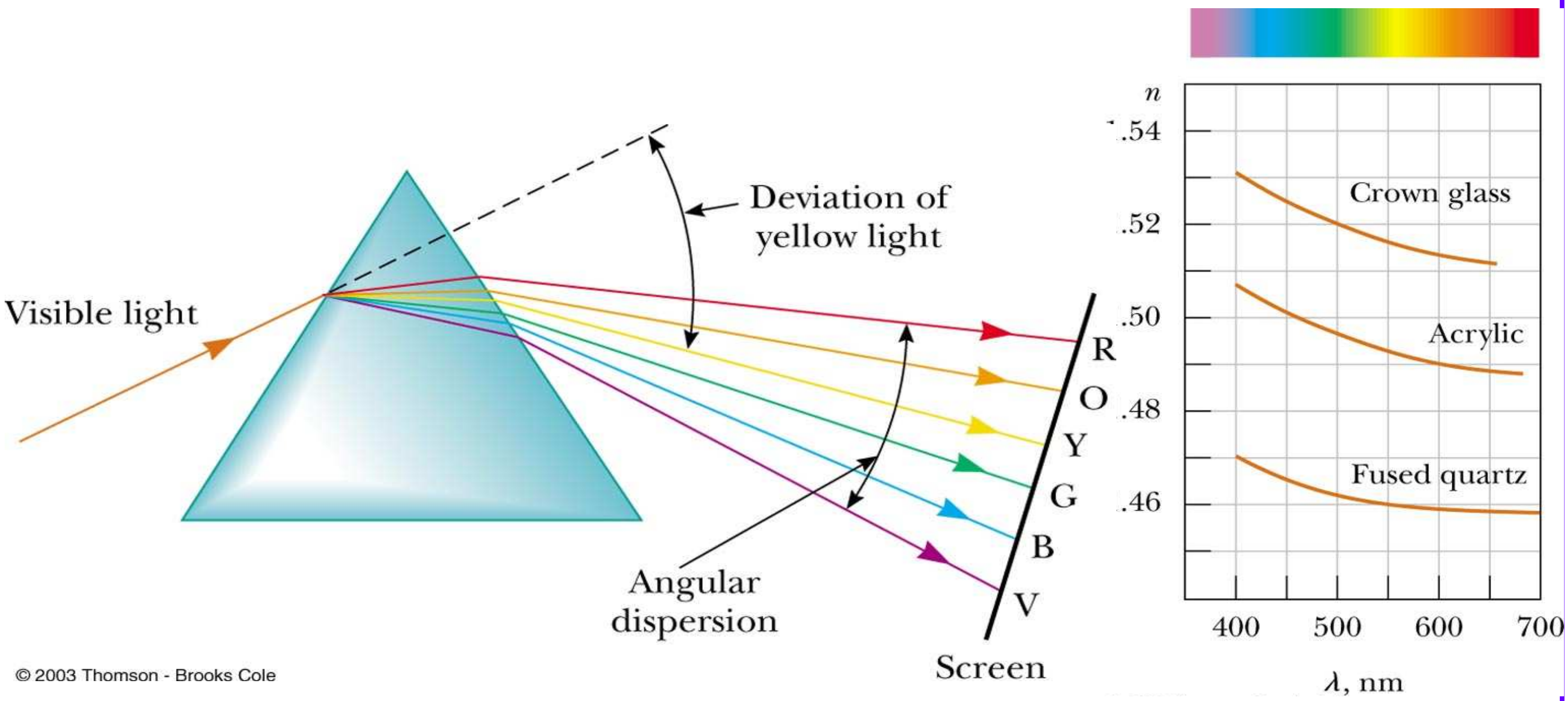
At high angles on incidence (grazing angles) only small differences of indices of refraction between the core and cladding are needed. They are made of two different kinds of glass.

Dispersion

- **The index of refraction in anything excep a vacuum depends on the wavelength of the light**
- **This dependence of n on λ is called *dispersion***
- **The index of refraction for a material usually decreases with increasing wavelength**
- **The angle of refraction when light enters a material depends on the wavelength of th light**
- **Violet light refracts more than red light when passing from air into a material**



Prism Dispersion



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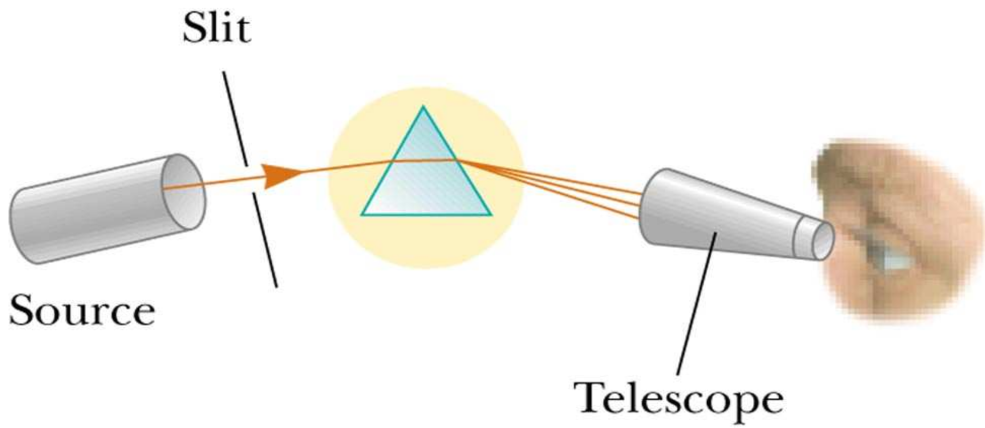
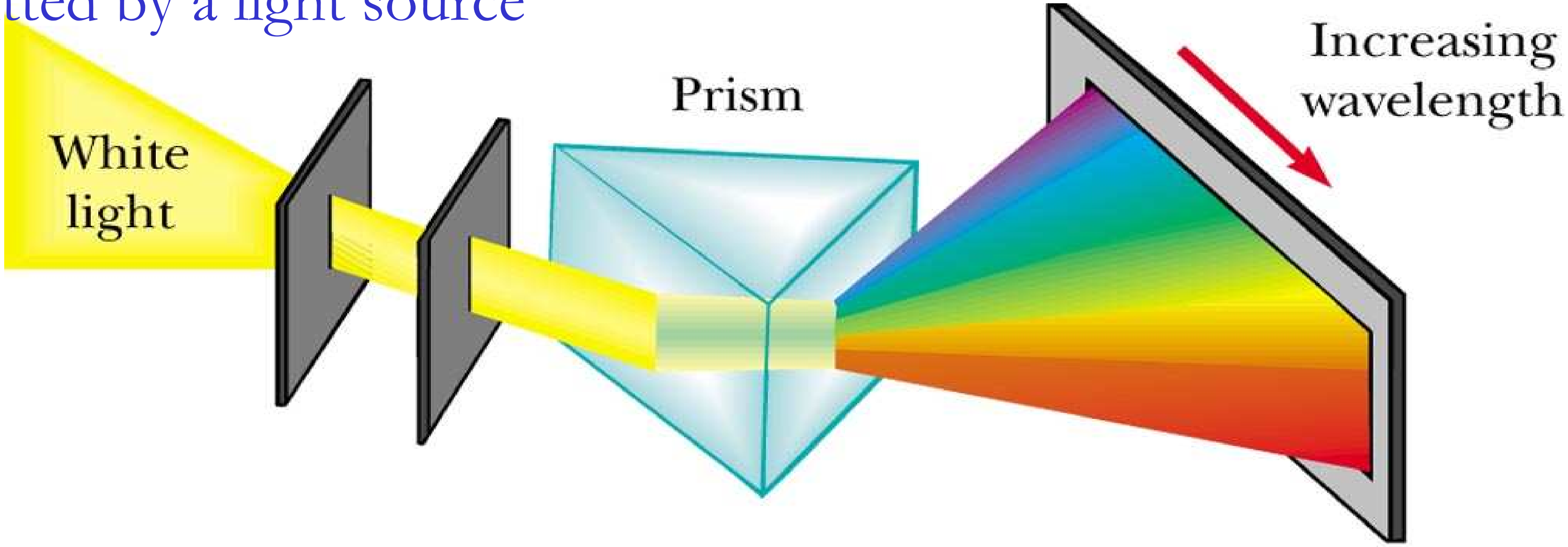
Prism Spectrometer

- A prism spectrometer uses a prism to cause the wavelengths to separate

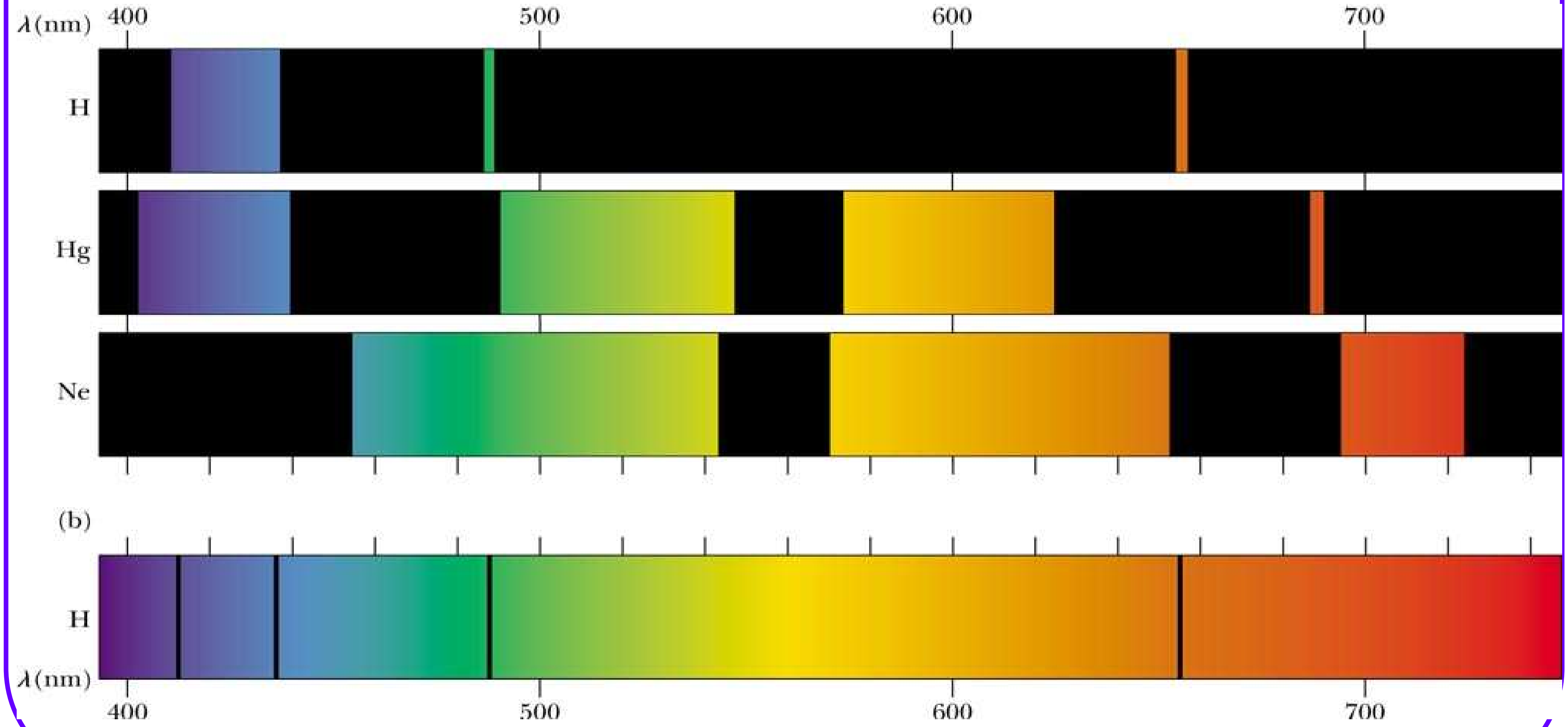


- The instrument is commonly used to study wavelengths emitted by a light source



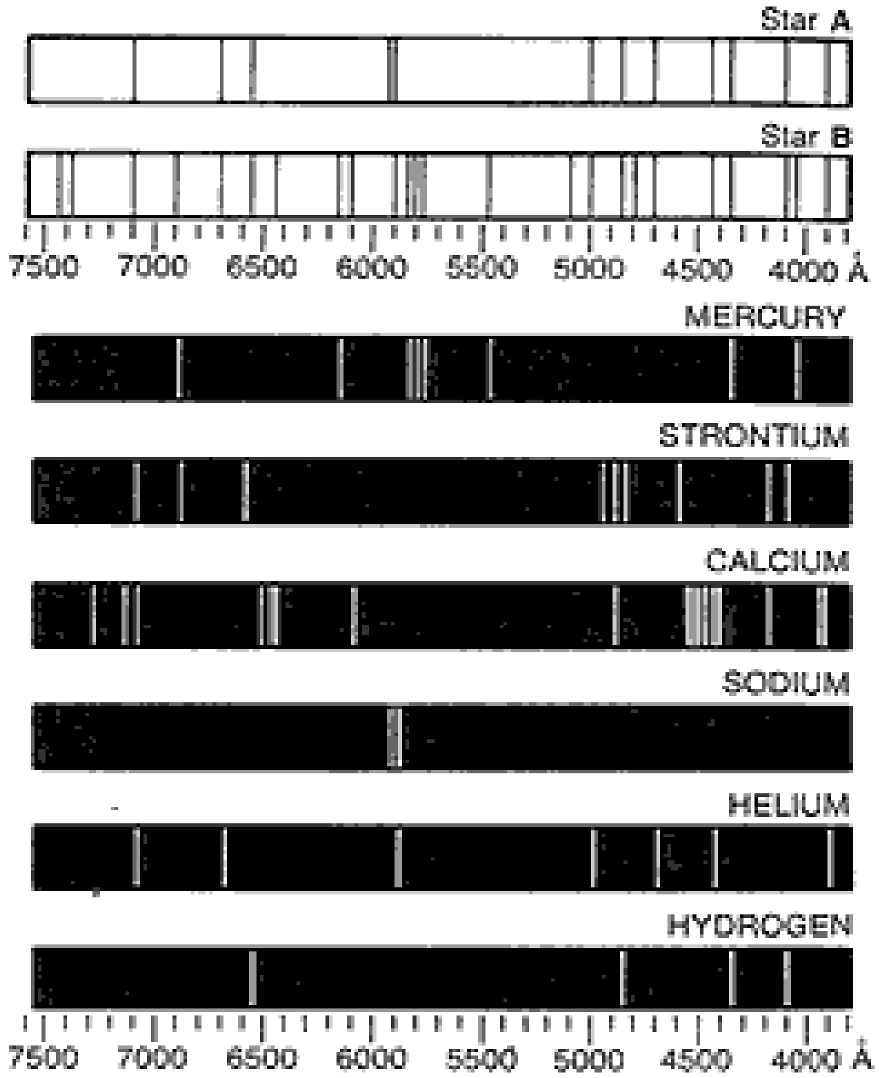
Examples of Spectra

Spectra of emission and absorption; line spectra and continuous spectra…

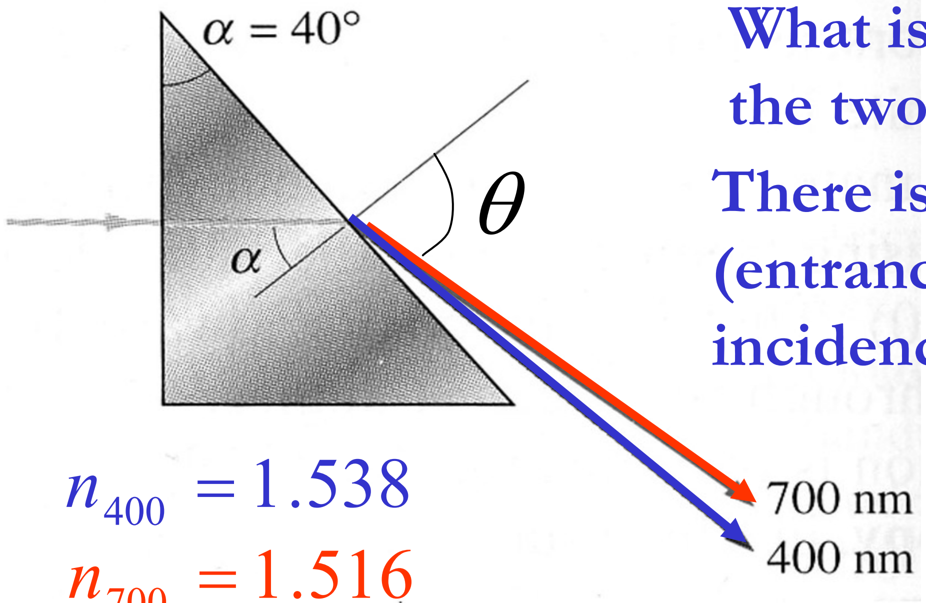


Using Spectra to Identify Gases

- All hot, low pressure gases emit their own characteristic spectra
- The particular wavelengths emitted by a gas serve as “fingerprints” of that gas
- Some uses of spectral analysis
  - Identification of molecules
  - Identification of elements in distant stars
  - Identification of minerals



Refraction in a Prism



$n_{400} = 1.538$

$n_{700} = 1.516$

What is the difference in the directions of the two beams, red and blue?

There is obviously no refraction at the left (entrance) interface, because the angle on incidence is 0.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Always satisfied if $\theta_1 = \theta_2 = 0$

Right interface – angle of incidence is $\alpha = 40°$.

Angles of refraction:

$$\theta_{700} = \sin^{-1}(n_{700} \sin 40°) = \sin^{-1}(1.516 \cdot 0.642) = 77.02°$$

$$\theta_{400} = \sin^{-1}(n_{400} \sin 40°) = \sin^{-1}(1.538 \cdot 0.642) = 81.34°$$

$$\theta_{400} - \theta_{700} = 4.32°$$