# Waves and Optics - PHY204
# (Smaldone - Sassi)

Gulu University

Naples FEDERICO II University

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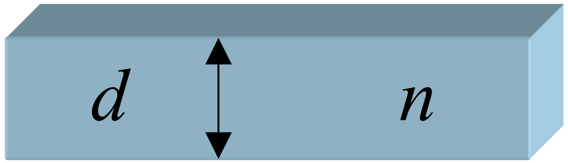
# Imaging

# Ray Optics

- **Basic Postulates:**
  1. **Light travel in form of rays. The rays are emitted by light sources and can be observed when they reach an optical detector;**
  2. **An optical medium is characterized by the refractive index n: n = c/v;**
     c: speed of light in free space; v: speed of light in the medium →



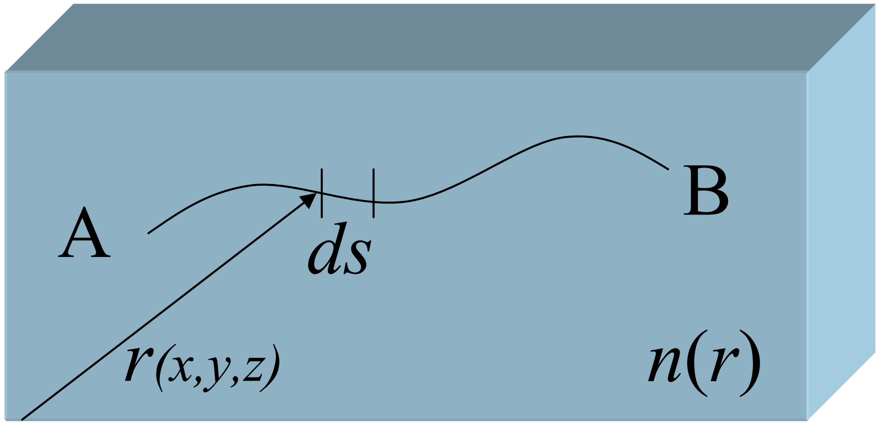

**Time to travel** = d/v= nd/*c*

=> **Optical path** = nd

# Basic Postulates (cont.)

**3. The optical path length along a given path between two points A and B in an inhomogeneous medium is:**

$$= \int_A^B n(r)\,ds$$




**4. Fermat's Principle: light rays travel along the path of least time:**

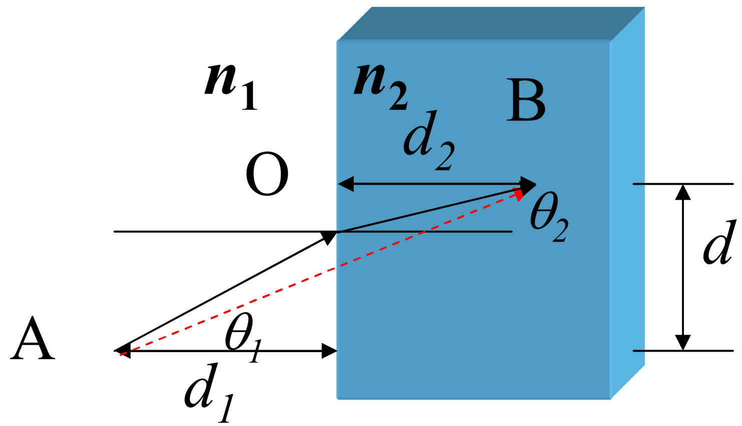
$$\text{minimum of} \left( \int_A^B n(r)\,ds \right)$$

**Could be more than one path !!!!**

# Application of Fermat's principle: Snell's law

**Minimize:**

$$\boxed{n_1\overline{AO}+n_2\overline{BO}=n_1\frac{d_1}{\cos\theta_1}+n_2\frac{d_2}{\cos\theta_2}}$$




**Under the constrain:**

$$d_1\tan\theta_1+d_2\tan\theta_2=d$$

Simplify the question by assuming $\mathbf{d_1=d_2}$

From the constrain $\rightarrow \tan\theta_1+\tan\theta_2=d/d_1 \Rightarrow$

$$\frac{d\tan\theta_1}{d\theta_1}+\frac{d\tan\theta_2}{d\theta_1}=0=\sec^2\theta_1+\sec^2\theta_2\frac{d\theta_2}{d\theta_1}=\frac{1}{\cos^2\theta_1}+\frac{1}{\cos^2\theta_2}\frac{d\theta_2}{d\theta_1}$$

$$\boxed{\Rightarrow\frac{d\theta_2}{d\theta_1}=-\frac{\cos^2\theta_2}{\cos^2\theta_1}}$$

# From Fermat's principle to Snell's law

$$n_1\overline{AO}+n_2\overline{BO}=n_1\frac{d_1}{\cos\theta_1}+n_2\frac{d_1}{\cos\theta_2}=\min\Rightarrow\qquad\frac{n_1}{\cos\theta_1}+\frac{n_2}{\cos\theta_2}=\min$$

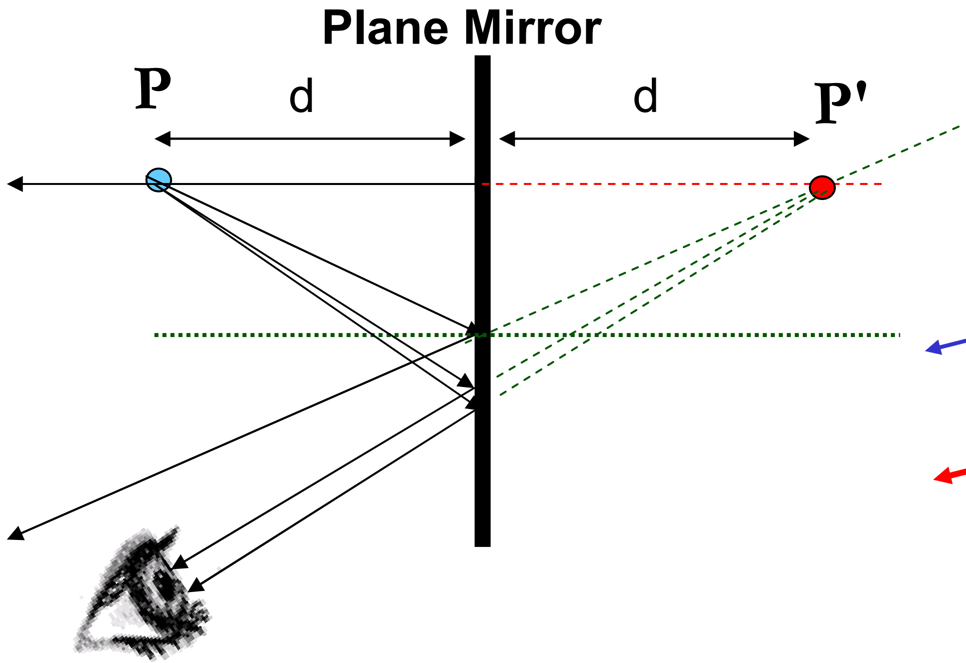
$$\therefore\frac{-n_1}{(\cos\theta_1)^2}\frac{d(\cos\theta_1)}{d\theta_1}+\frac{-n_2}{\cos^2\theta_2}\frac{d(\cos\theta_2)}{d\theta_1}=0=\frac{n_1\sin\theta_1}{\cos^2\theta_1}+\frac{n_2\sin\theta_2}{\cos^2\theta_2}\frac{d\theta_2}{d\theta_1}$$

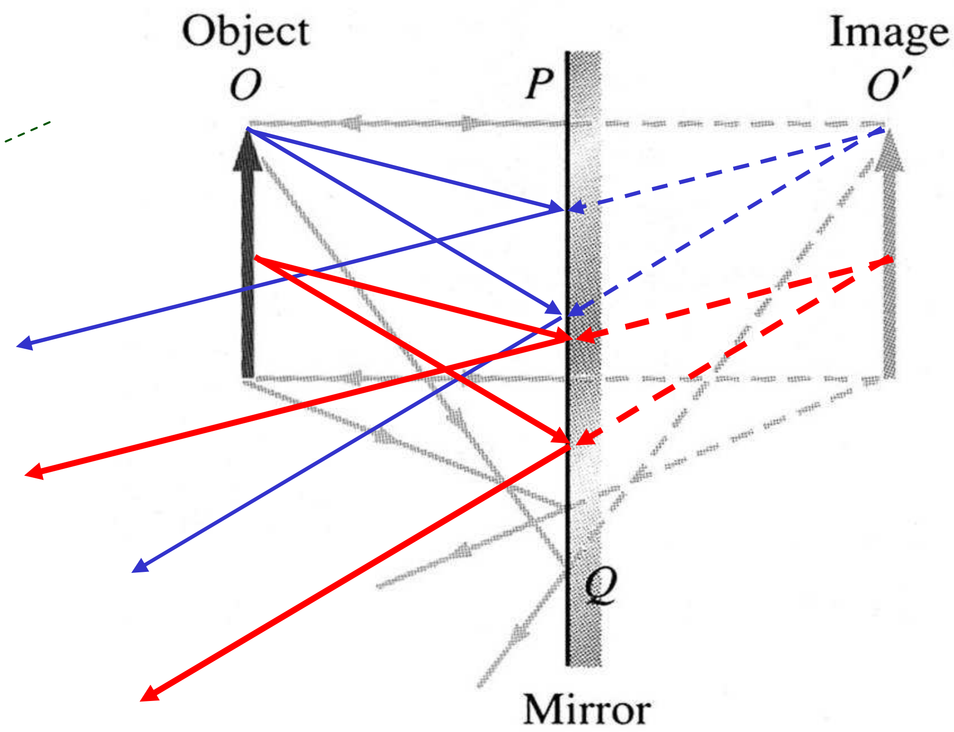
$$\therefore\frac{n_1\sin\theta_1}{\cos^2\theta_1}+\frac{n_2\sin\theta_2}{\cos^2\theta_2}\left(-\frac{\cos^2\theta_2}{\cos^2\theta_1}\right)=\frac{n_1\sin\theta_1-n_2\sin\theta_2}{\cos^2\theta_1}=0$$

Minimize optical path => $\boxed{n_1\sin\theta_1=n_2\sin\theta_2}$

Snell's Law !!

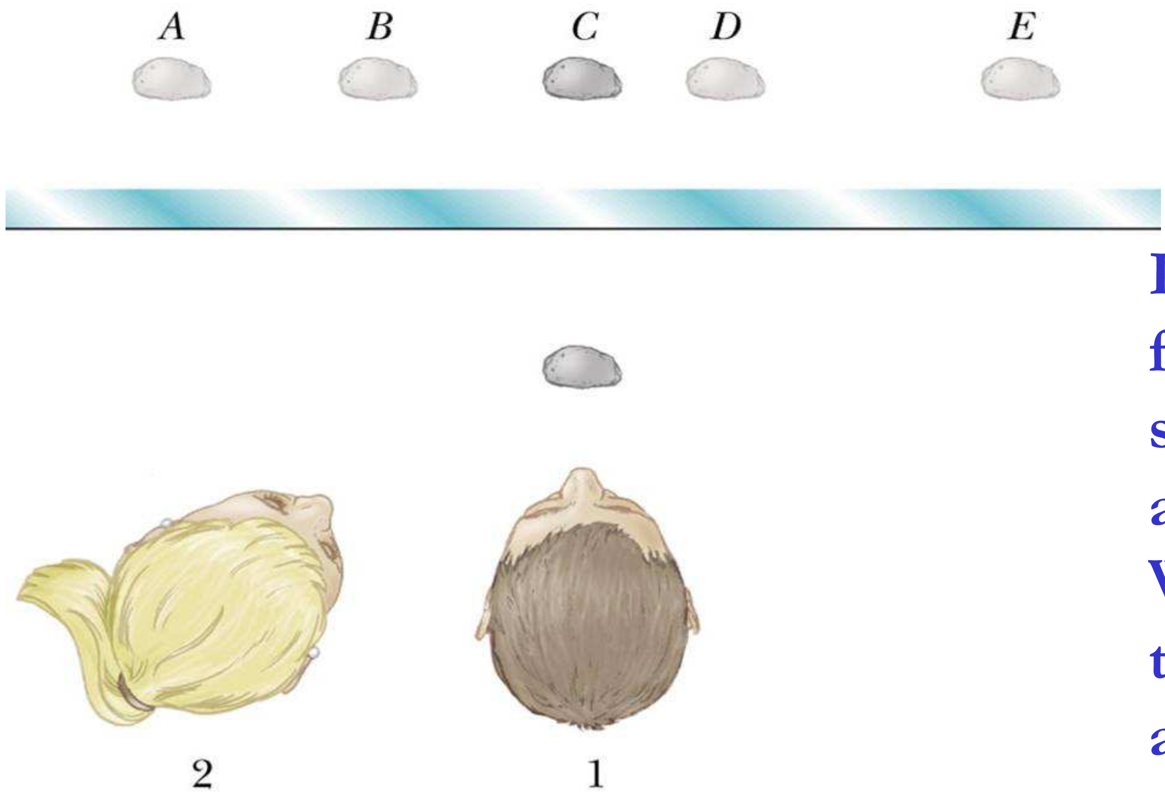
# Image Formation in a Plane Mirror




The image is always there, in a well defined position, whether you look at it or not.
The image is formed behind the mirror at a distance equal to the distance from the object, and has size equal to the size of the object.
It is a "*virtual*" image, since there is no light in the image location. The light only appears to come from there.

# Plane Mirror Exercise 1

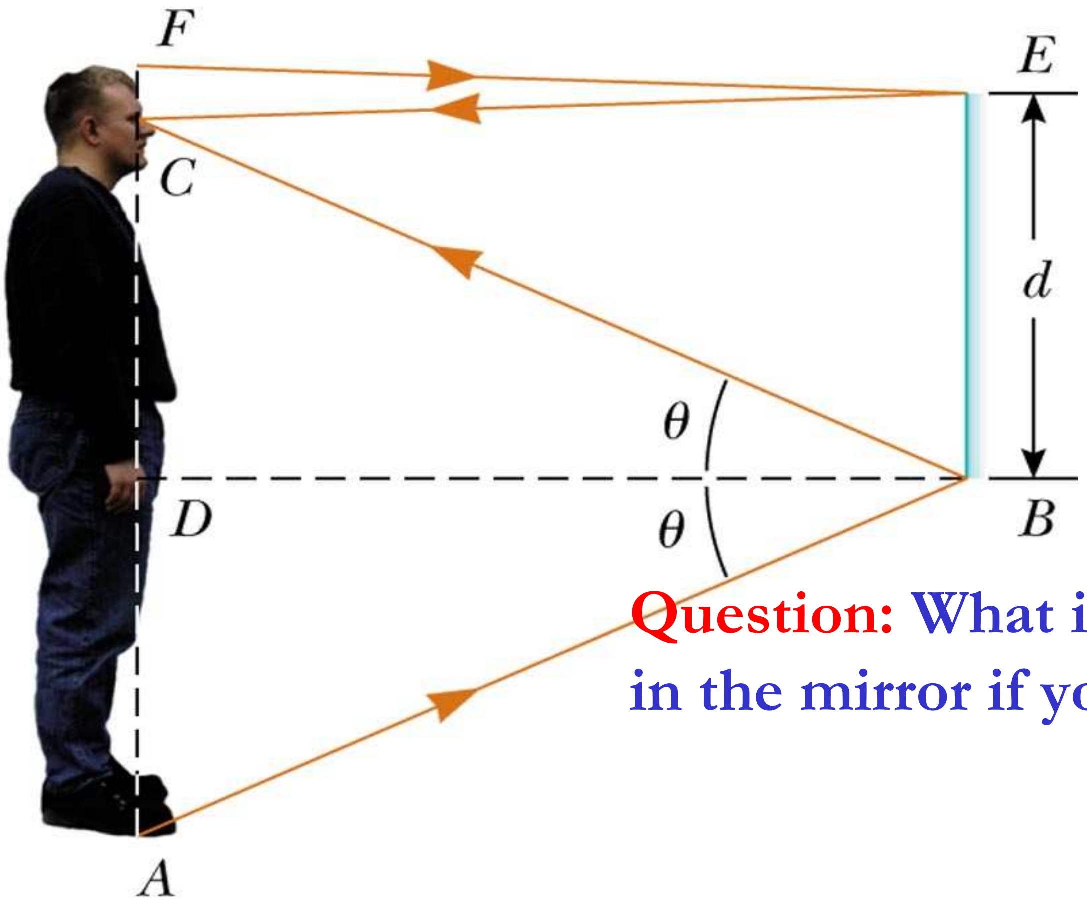



←Plane Mirror

In the overhead view of the figure, the image of the stone seen by observer **1** is at $C$.
Where does observer **2** see the image – at $A$, at $B$, at $C$, at $D$, at $E$, or not at all?

**Position of an image is defined just as well as position of the object! A little geometry demonstrates that its location is independent of the position of the observer!**

# Looking into a Mirror

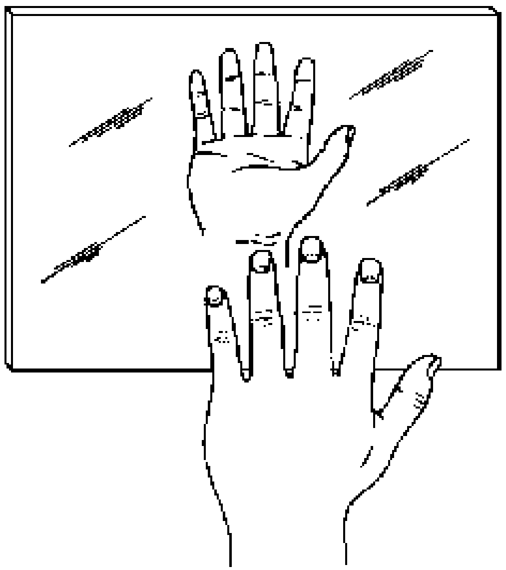



Looking into a mirror at yourself: for an unobstructed, complete view you only need a mirror, which is a half of your height.

**Question:** What is going to happen to your image in the mirror if you walk away from it?

**Answer:** Nothing other than it will appear to be further away, twice the distance of that from you to the mirror.

# "a Mirror Image"

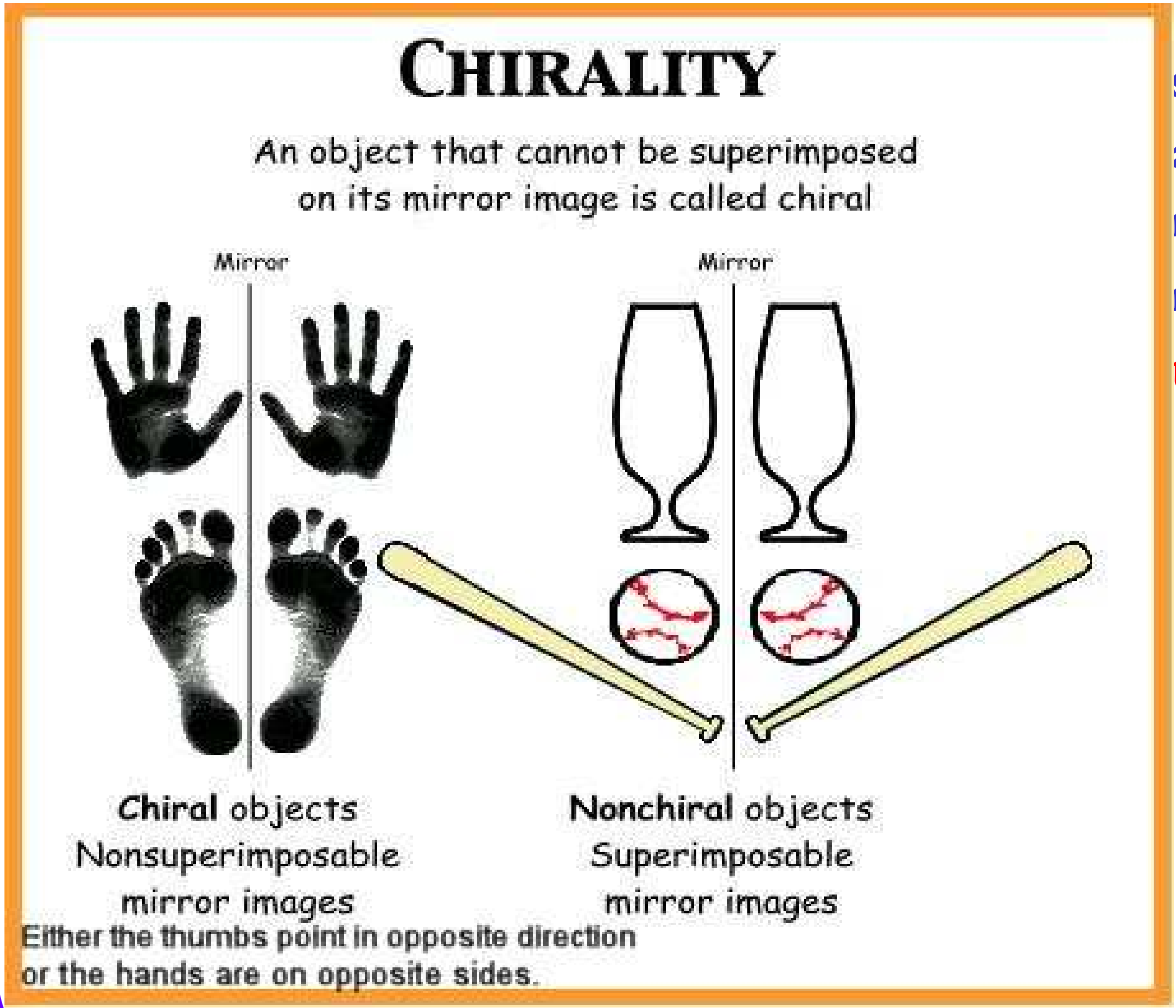


**Mirrors are known to turn left into right, that is to make the image of your left hand look as your right hand.**

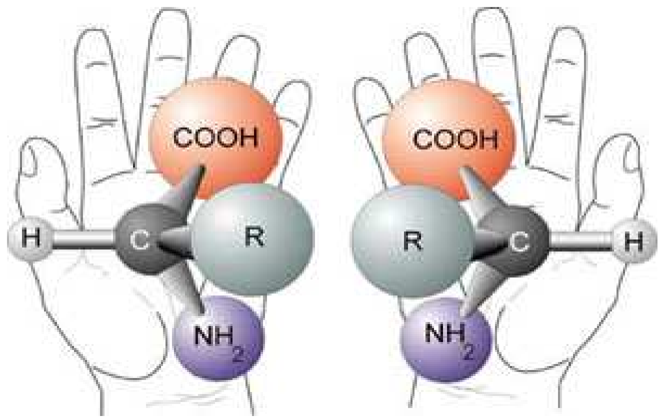
**It is this effect that gives rise to the expression**

**"A Mirror Image"**

# Chirality

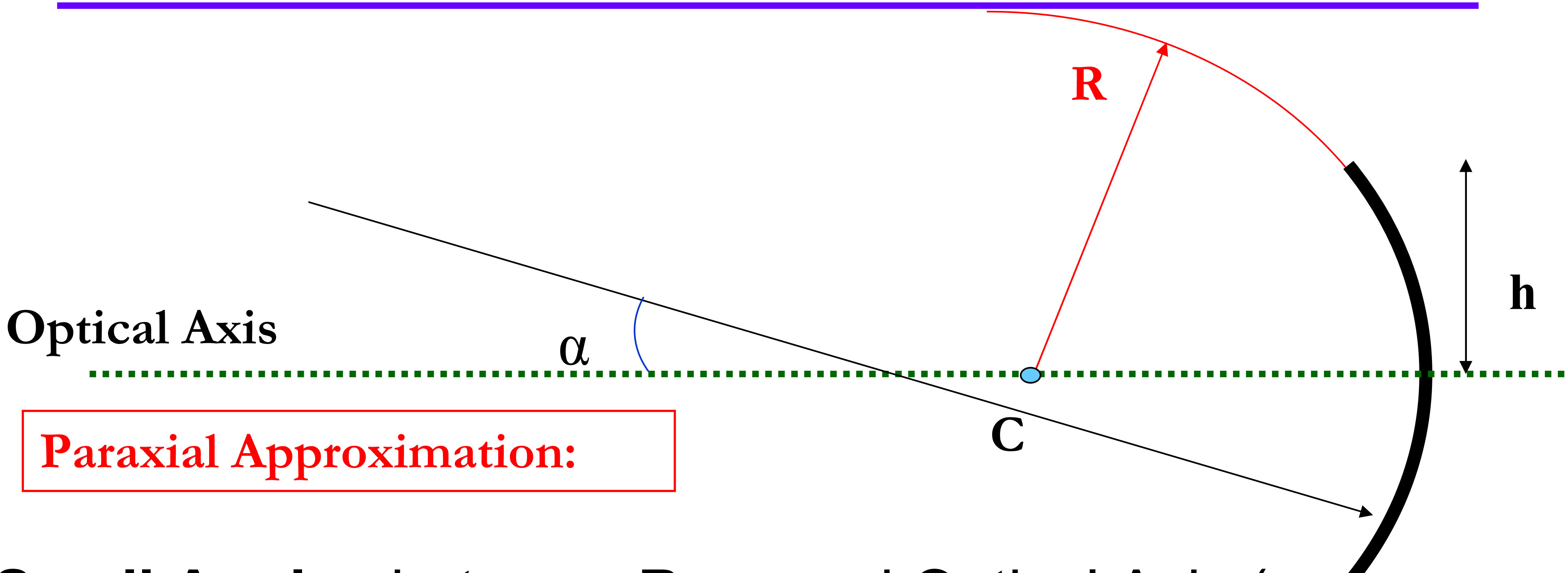



The mirrors actually do a very special transformation, known as ***inversion***, which cannot be reduced to translations and rotations… Maybe to turning inside out? ...




Chiral objects and chiral molecules…

# "Axial" Optical Systems

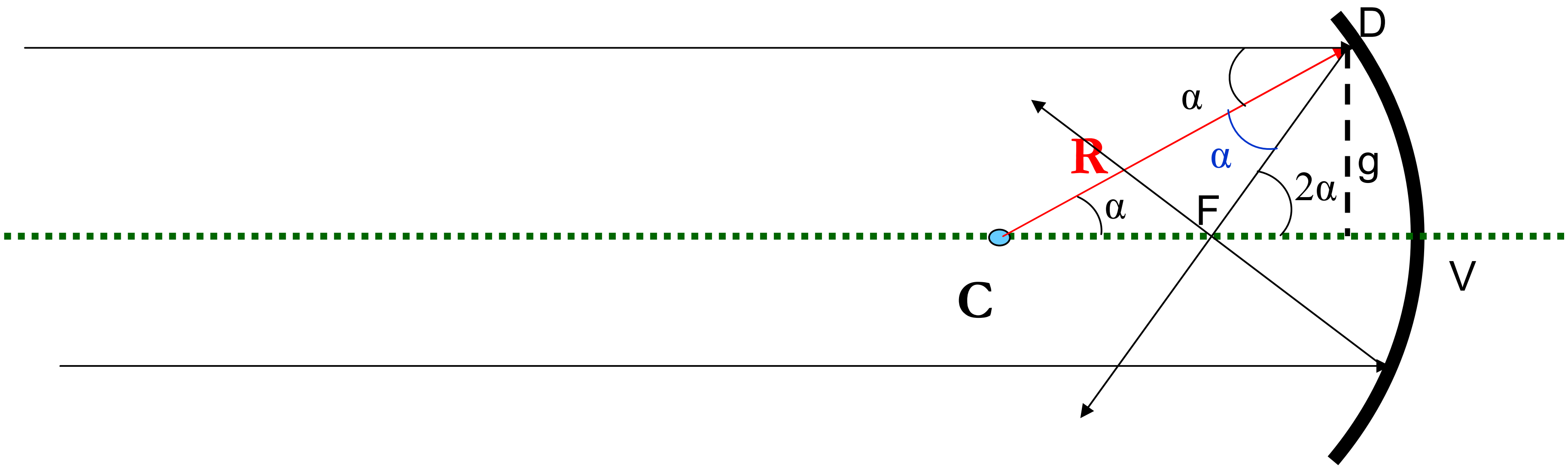



**Paraxial Approximation:**

- **Small Angles** between Rays and Optical Axis ( $\boldsymbol{\alpha \approx \sin\alpha \approx \tan\alpha}$ $\Rightarrow$ Snell's Law: $n_1\theta_1 = n_2\theta_2$)
- **Linear Characteristic Dimensions** of surfaces **small** comparing with curvature radii ( $\mathbf{h \ll R}$ )

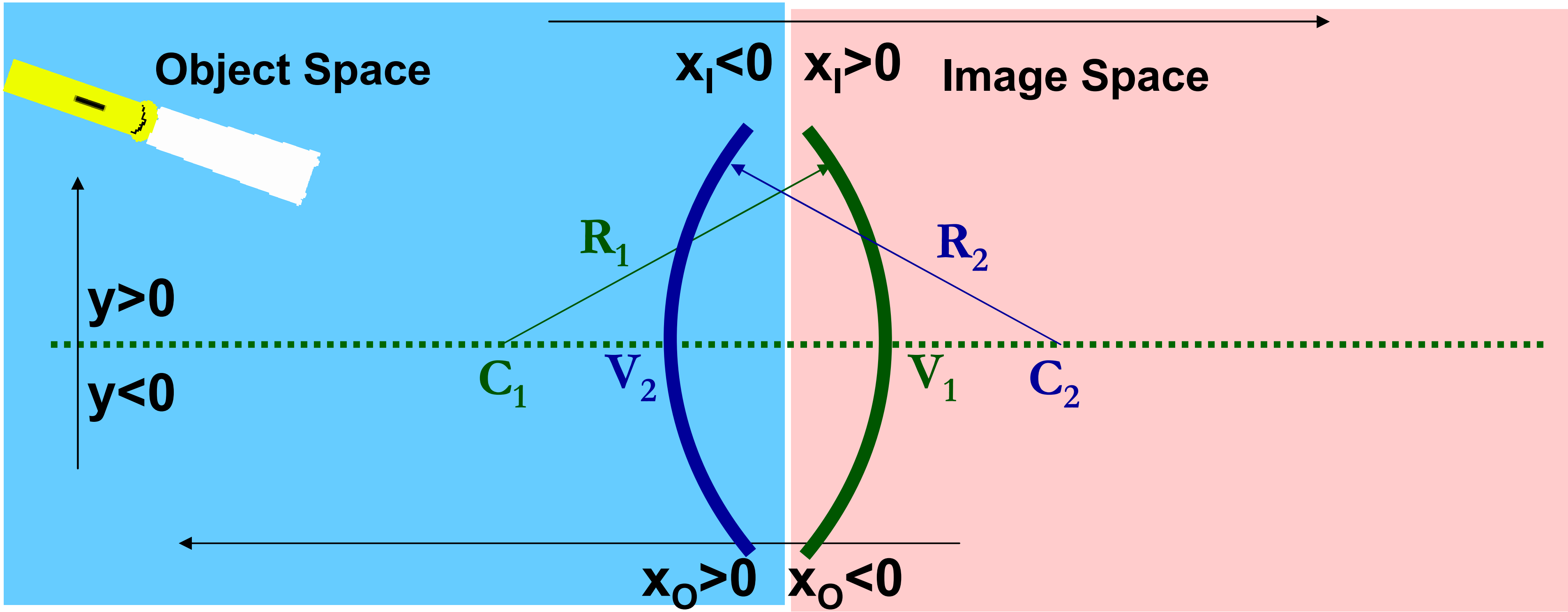
# Focal Point of a Spherical Mirror

$\underline{CF}=\underline{FD} \quad g=R\alpha=2\alpha\cdot\underline{FD} \Rightarrow \underline{FD}=R/2 \Rightarrow \underline{CF}=\underline{FV}=R/2=f$




- Rays parallel to Optical Axis → focal point F;
- Ray from center of curvature C → C
- A point light source in F → Rays parallel to Optical Axis

# (my) Signs Convention



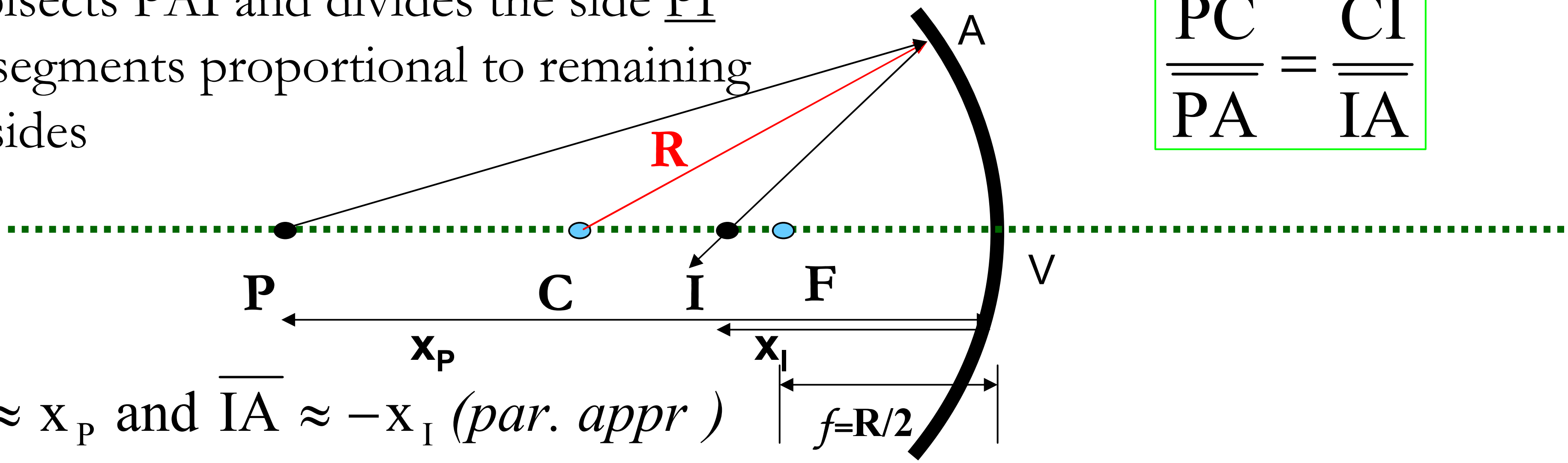

| Refracting Surfaces | |
|---|---|
| $R_1$ | - if $C_1$ is left of $V_1$ |
| $R_2$ | + if $C_2$ is right of $V_2$ |

| Reflecting Surfaces | |
|---|---|
| $R_1$ | - if $C_1$ is left of $V_1$ |
| $R_2$ | + if $C_2$ is right of $V_2$ |

# Spherical Mirror Equation

CA bisects PAI and divides the side PI into segments proportional to remaining two sides

$$\frac{\overline{PC}}{\overline{PA}} = \frac{\overline{CI}}{\overline{IA}}$$




$\overline{PA} \approx x_P$ and $\overline{IA} \approx -x_I$ *(par. appr )*

$$\overline{PC} = x_P - |R|$$
$$\overline{CI} = |R| - |x_I|$$

**Sign conv. |R|=-R**

$$\overline{PC} = x_P + R$$
$$\overline{CI} = -R + x_I$$

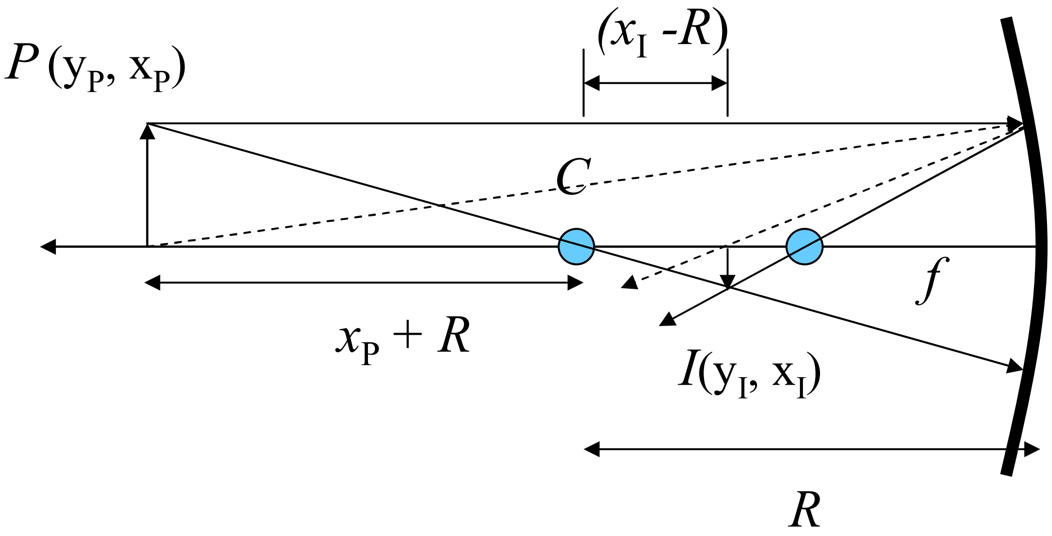
$$\frac{x_P + R}{x_P} = \frac{-R + x_I}{-x_I} \Rightarrow \boxed{\frac{1}{x_P} - \frac{1}{x_I} = -\frac{2}{R} = \frac{1}{f}}$$

*($f$ > 0!!)*

# Spherical Mirror Image Formation

$$\frac{y_P}{y_I} = \frac{x_P + R}{-(x_I - R)}$$

From $$\frac{1}{x_P} - \frac{1}{x_I} = -\frac{2}{R}$$



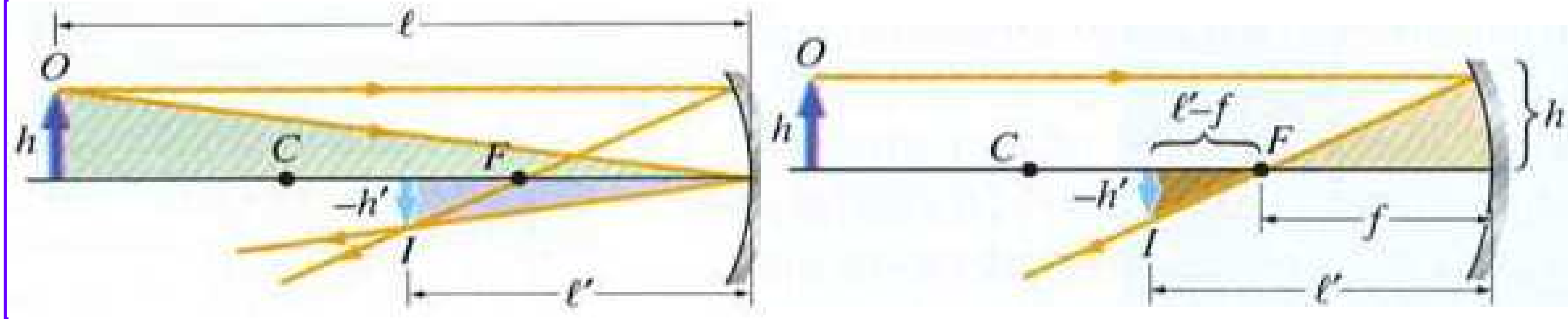

$$\frac{1}{x_P} + \frac{1}{R} = -\frac{1}{R} + \frac{1}{x_I} = \frac{R + x_P}{x_P R} = \frac{R - x_I}{x_I R} \Rightarrow \boxed{\frac{x_P + R}{R - x_I} = \frac{x_P}{x_I}}$$

$$\boxed{\frac{y_P}{y_I} = \frac{x_P}{x_I} \Rightarrow y_I = \frac{x_I}{x_P} y_P = M y_P}$$

Where $M = x_I / x_P$ is **magnification**

**(If M<0 : inverted !)**

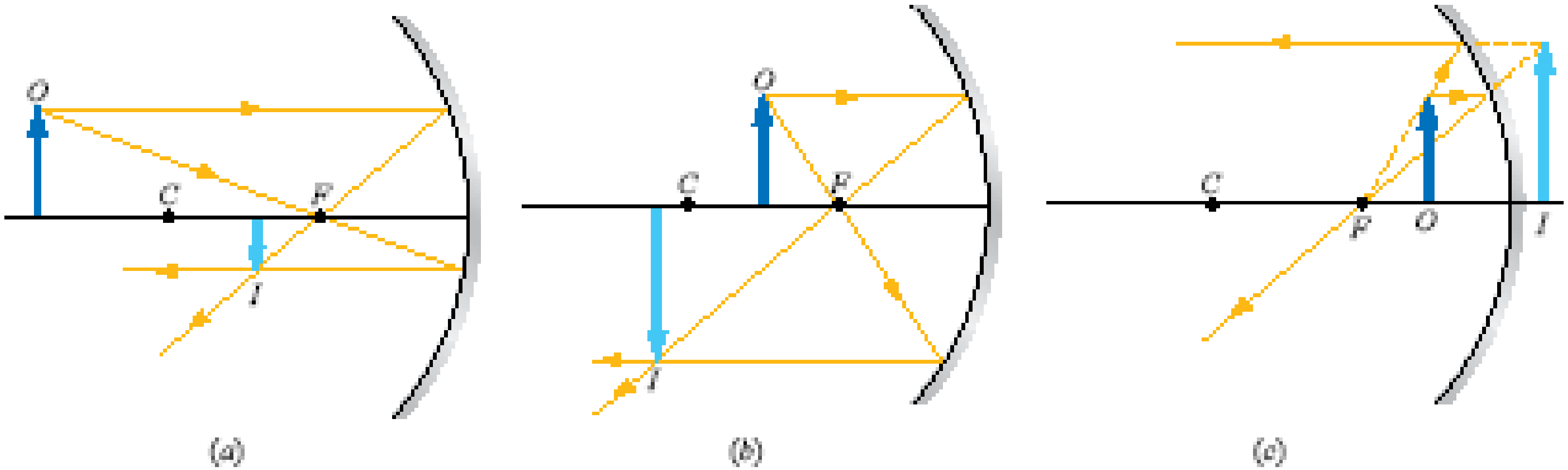
# To Help to Locate Image (Spherical Mirrors)




1. Any ray parallel to mirror axis reflects through the focal point
2. Any ray that passes through the focal point reflects parallel to the axis
3. Any ray that strikes the center of the mirror reflects symmetrically about the mirror axis
4. Any ray that that passes through the center of curvature returns on itself

**Any two of these rays is sufficient to locate an image**

# Images from Concave Spherical Mirrors



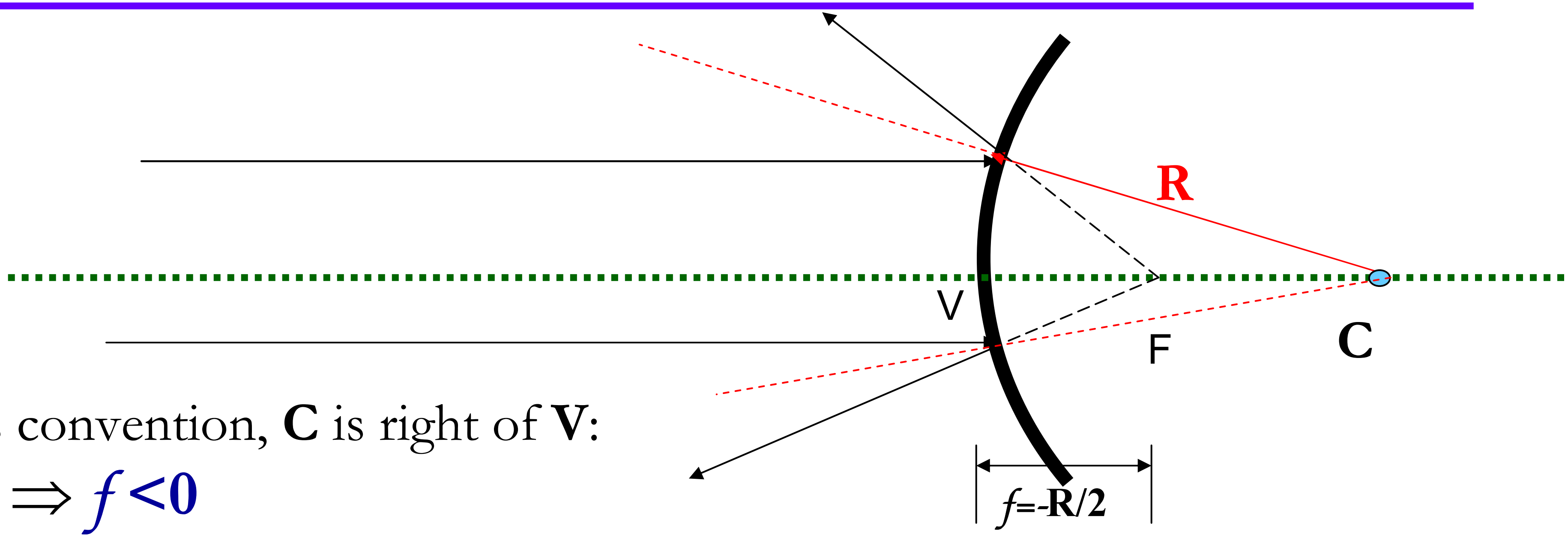

**Concave Spherical Mirrors form images:**

$$x_I = \frac{x_P f}{f - x_P} \quad \therefore \; f = -\frac{R}{2} > 0$$

$$M = \frac{x_I}{x_P}$$

**(a) Object beyond C, image is real, reduced, and inverted.**
**(b) Object between C and F, image is real, magnified, and inverted.**
**(c) Object inside F, image is virtual, magnified, and upright.**

# Convex Spherical Mirrors




Signs convention, **C** is right of **V**:

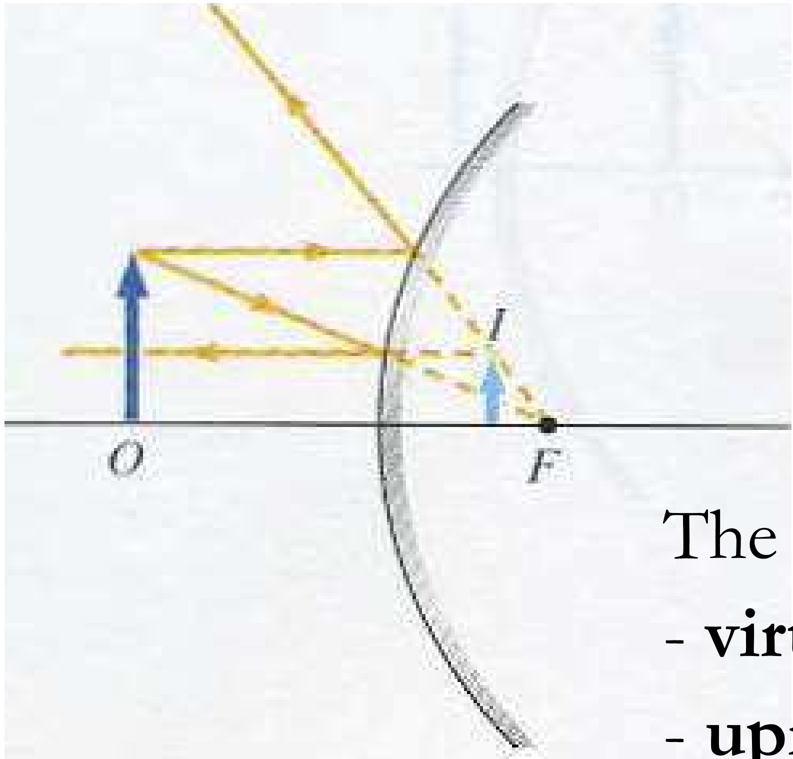
**R + $\Rightarrow f<0$**

$$\boxed{\frac{1}{x_P}-\frac{1}{x_I}=\frac{1}{f}} \qquad \Rightarrow \frac{1}{x_I}=\frac{1}{x_P}-\frac{1}{f}>0$$

**virtual image** (in the back side of mirror)

**$M=x_I/x_P$ : $M>0$** (**upright**) and **$M<1$** (**reduced**)

# Image Using Convex Mirror

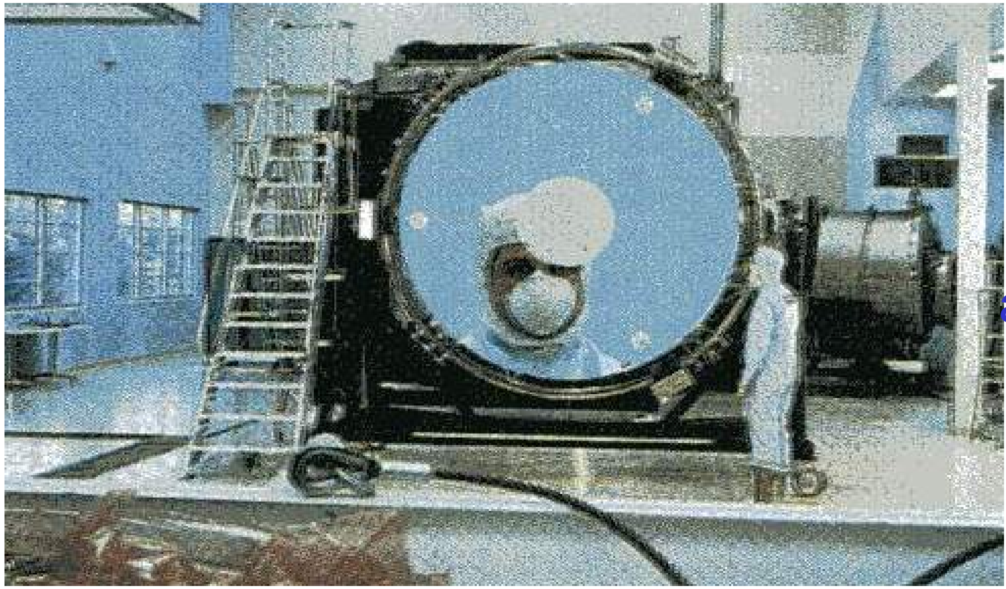



$$x_I = \frac{x_P f}{f - x_P} \quad \therefore \; f = -\frac{R}{2} < 0$$

The image is always:

- **virtual** (**$x_I$>0**, on the mirror back)
- **upright** (**$x_I$>0**, **$x_P$>0** , **M>0**)
- **reduced** in size (**$x_I$<$x_P$** , **M<1**)

# Concave Mirror Exercise



A technician stands 3.85m (from the vertex) in front of concave mirror which has a focal length of 5.52m.

a) The **location** of the tech's image is?

b) Is tech's image **virtual**?

c) What is its **magnification** ?

d) What is the curvature **radius** of the mirror?

The technician moves to 15.00 m from the concave mirror. Answer again the questions a,b and c.

$$\boxed{\frac{1}{x_P} - \frac{1}{x_I} = \frac{1}{f}} \qquad \boxed{M = \frac{x_I}{x_P}}$$

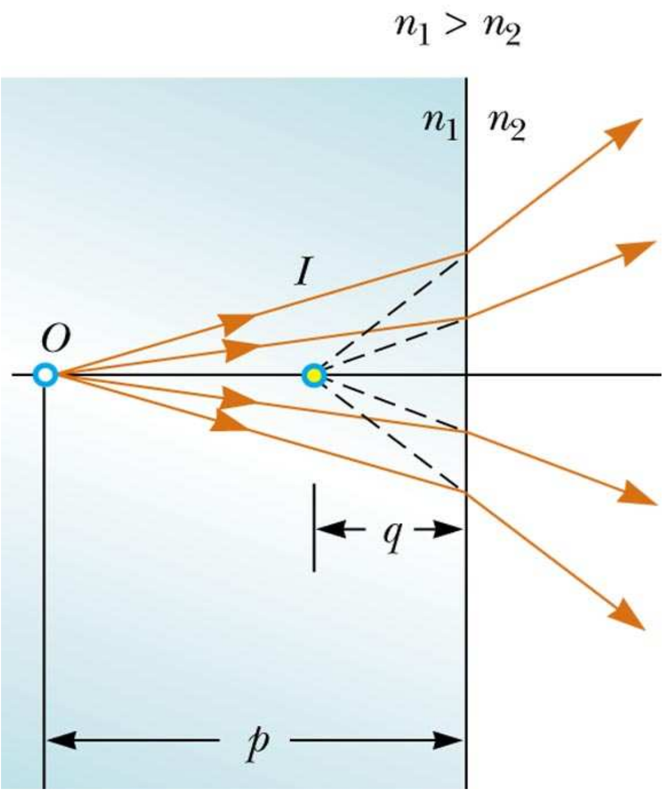
$$\frac{1}{x_I} = \frac{1}{3.85} - \frac{1}{5.52} = 0.0786 \Rightarrow x_I = +12.72m$$

(back to the mirror ⇒ virtual image)

$M = x_I/x_P = +12.72/3.85 = +3.30$, upright $\quad R = 2f = 11.04$ m $\quad$ [ 8.74 m; real; -0.58]

# Do you need a mirror to form an image?

**Not necessarily. You can do reasonably well with a flat refracting surface.**

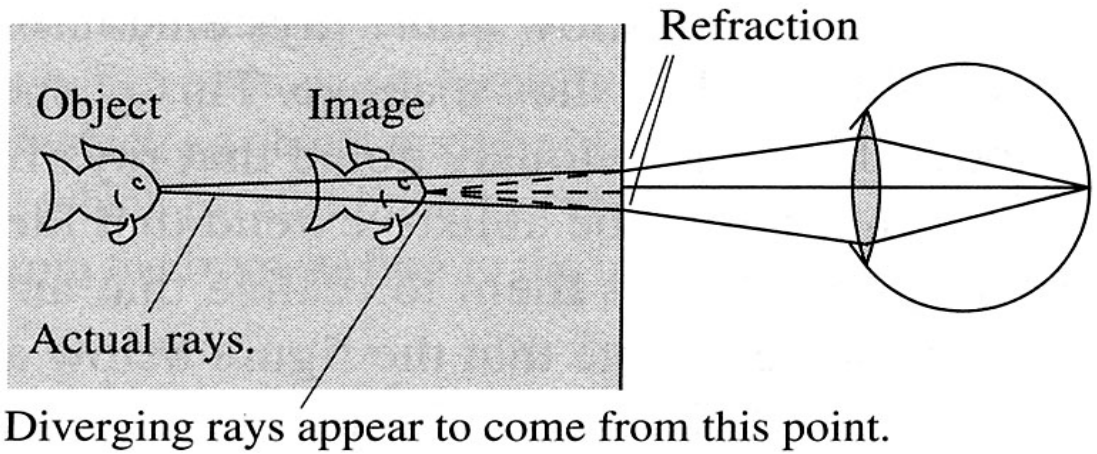



The image formed by a flat refracting surface is on the ***same*** side of the surface as the object

The image is ***virtual***

The image forms between the object and the surface

The rays bend away from the normal since $n_1 > n_2$

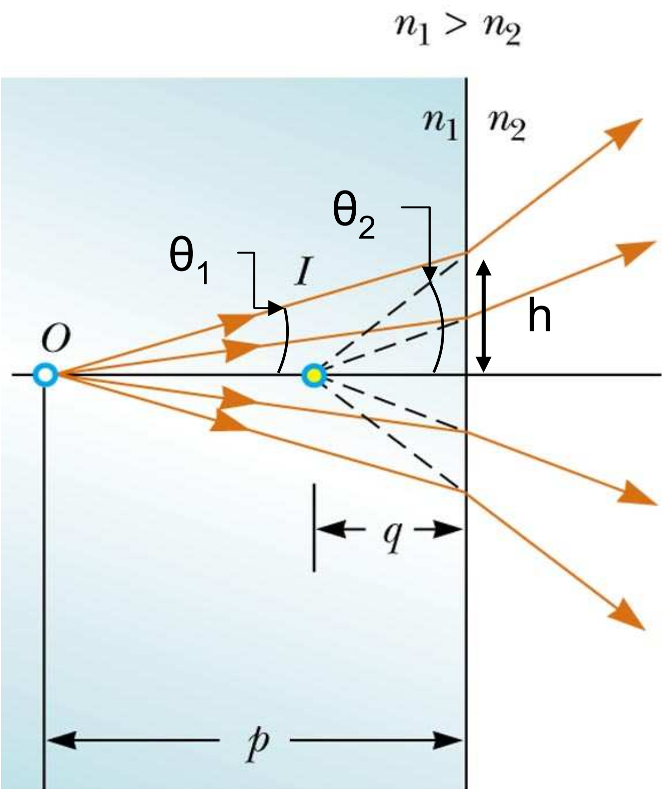


# Depth perception looking into water

The object is at *O*, the image is at *I*

Assume a light ray from the object has an incident angle of $\theta_1 \ll 1$

Assume light ray from the Image has an incident angle of $\theta_2 \ll 1$




**If the depth of a pool is *p*=2 *m* what is the perception of the pool depth, q ?**

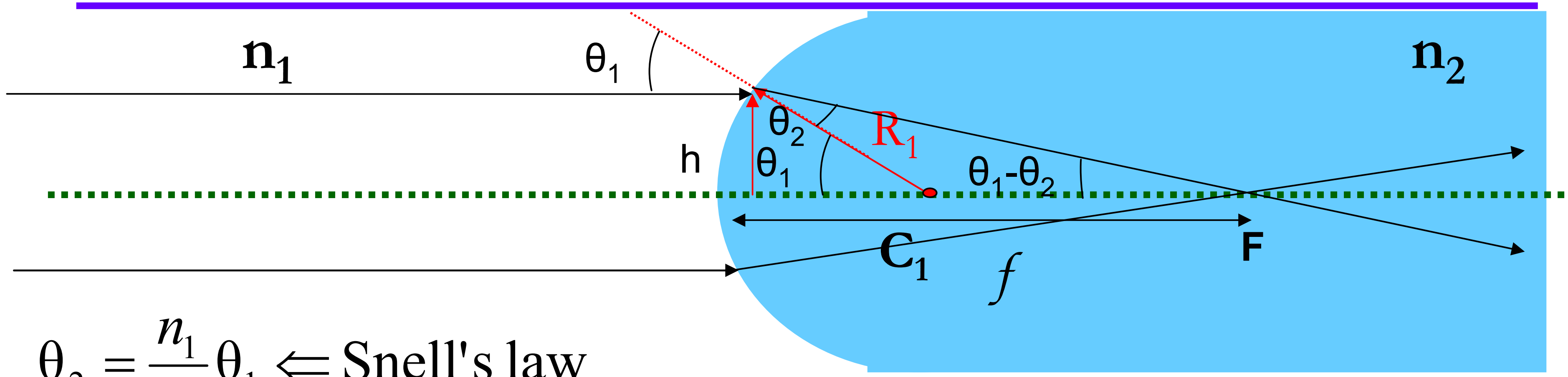
$$\sin\theta_1 \approx \tan\theta_1 \approx \theta_1 \approx \mathrm{h/p}$$

$$\sin\theta_2 \approx \tan\theta_2 \approx \theta_2 \approx \mathrm{h/q}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{and} \quad n_1 = 1.33 = 4/3,\ n_2 = 1$$

$$\boxed{\frac{4}{3}\frac{h}{p} \approx \frac{h}{q} \Rightarrow \mathrm{q} = \frac{3}{4}p = 1.5m}$$

# Spherical Interface




$$\theta_2 = \frac{n_1}{n_2}\theta_1 \Leftarrow \text{Snell's law}$$

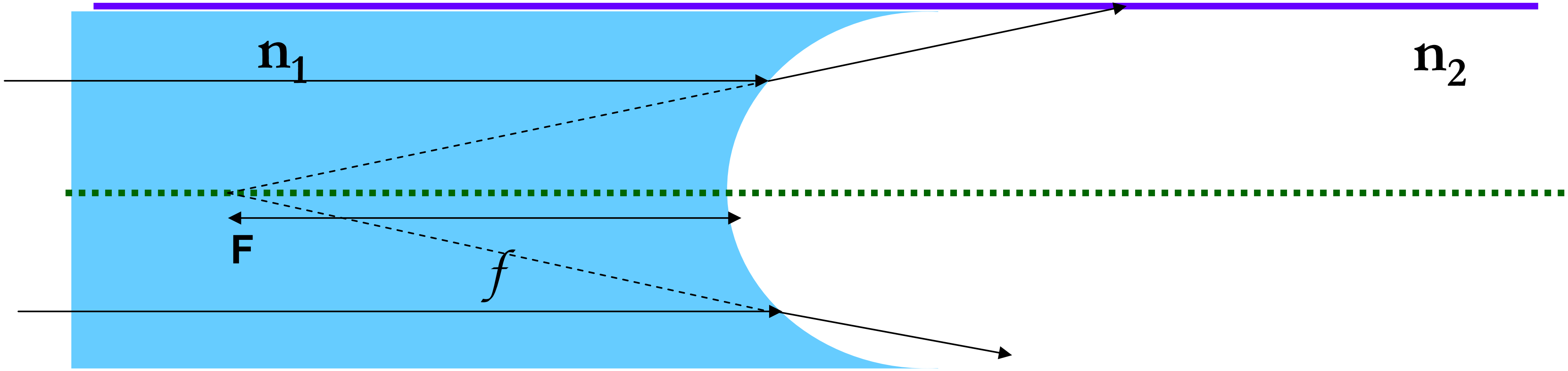
$$h = R_1\theta_1$$

$$f = \frac{h}{\theta_1 - \theta_2} = \frac{R_1\theta_1}{\theta_1\left(1 - \frac{n_1}{n_2}\right)} \Rightarrow$$

$$\boxed{f = \frac{n_2 R_1}{n_2 - n_1}}$$

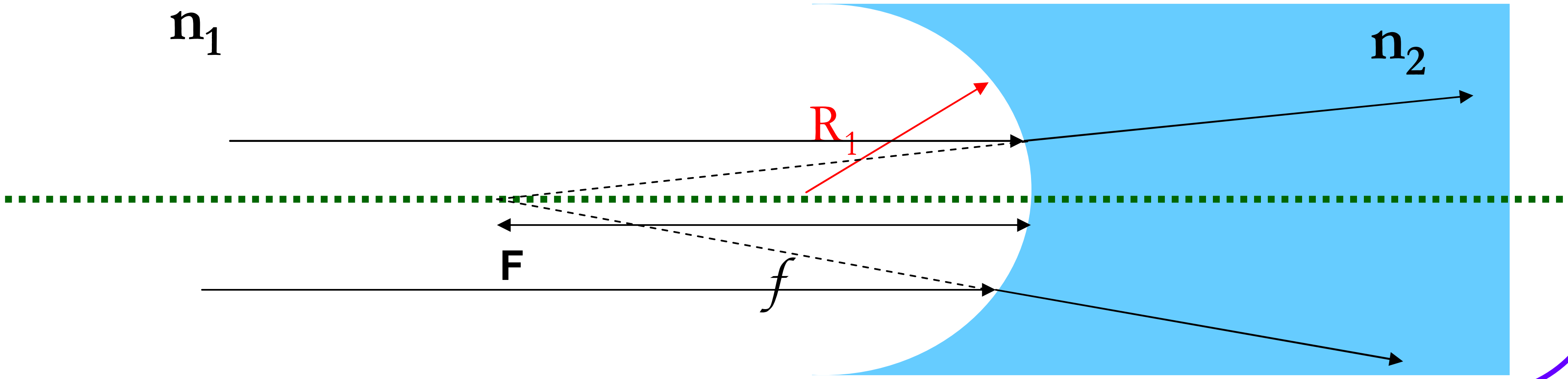
If $\mathbf{R_1>0}$ (as in figure) $\Rightarrow f\mathbf{>0}$ if $\mathbf{n_2>n_1}$ (as in figure)

# Spherical Interface - 2



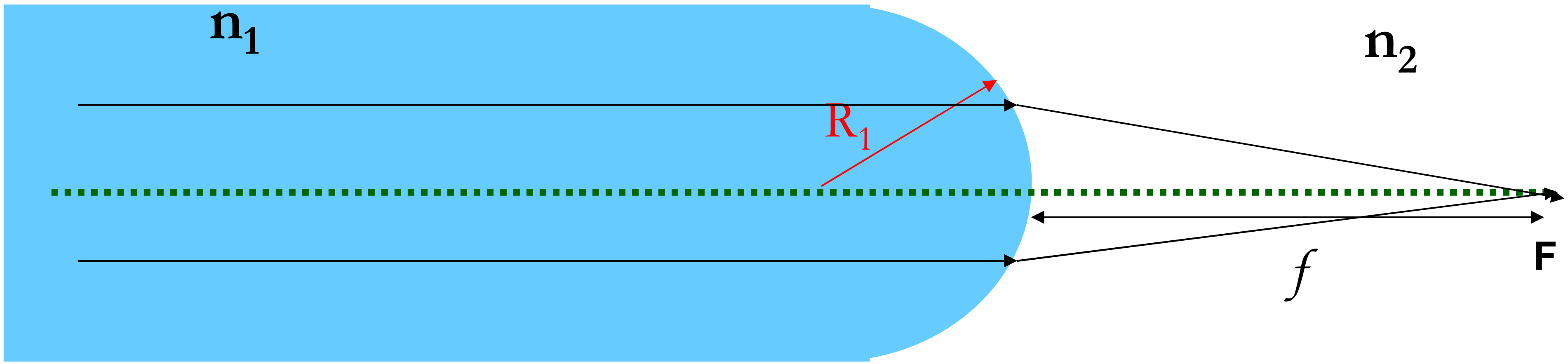

If $\mathbf{R_1>0}$ (as before) $\Rightarrow f\mathbf{<0}$ if $\mathbf{n_2<n_1}$ (as top figure)

If $\mathbf{R_1<0}$ (as in the bottom figure) $\Rightarrow f\mathbf{<0}$ if $\mathbf{n_2>n_1}$

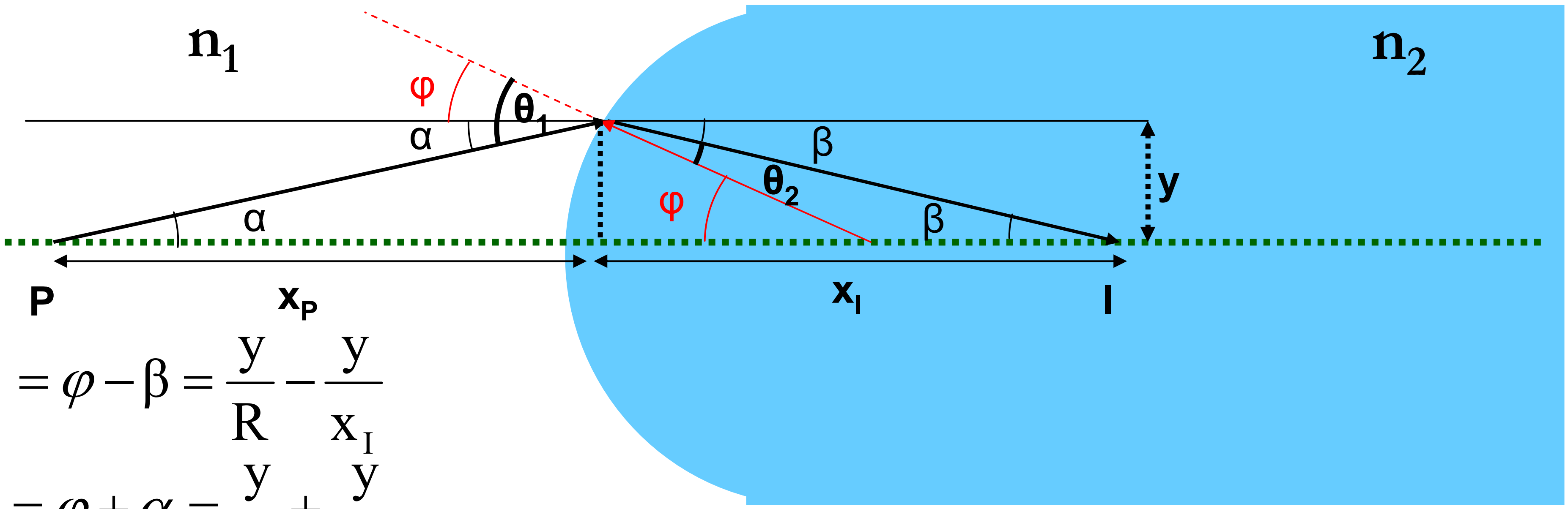


# Spherical Interface -3

If $\mathbf{R_1<0}$ (as in the bottom figure) $\Rightarrow f\mathbf{>0}$ if $\mathbf{n_2<n_1}$



# Image Formation by Spherical Interface



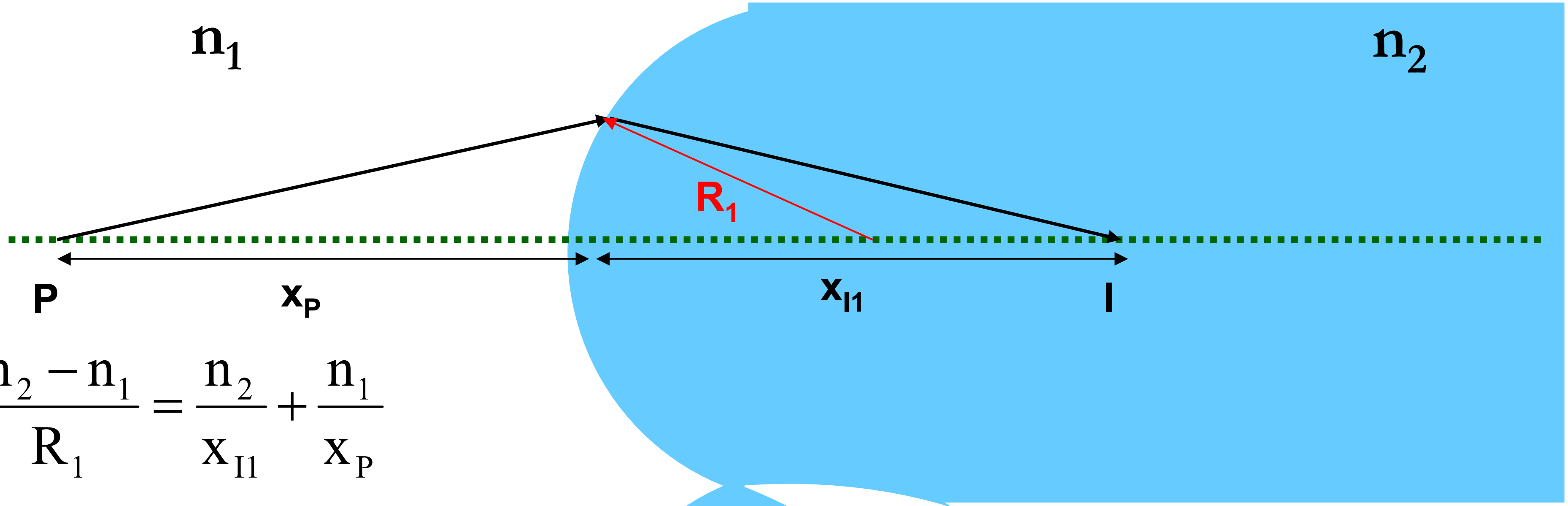

$$\theta_2 = \varphi - \beta = \frac{y}{R} - \frac{y}{x_I}$$

$$\theta_1 = \varphi + \alpha = \frac{y}{R} + \frac{y}{x_P}$$

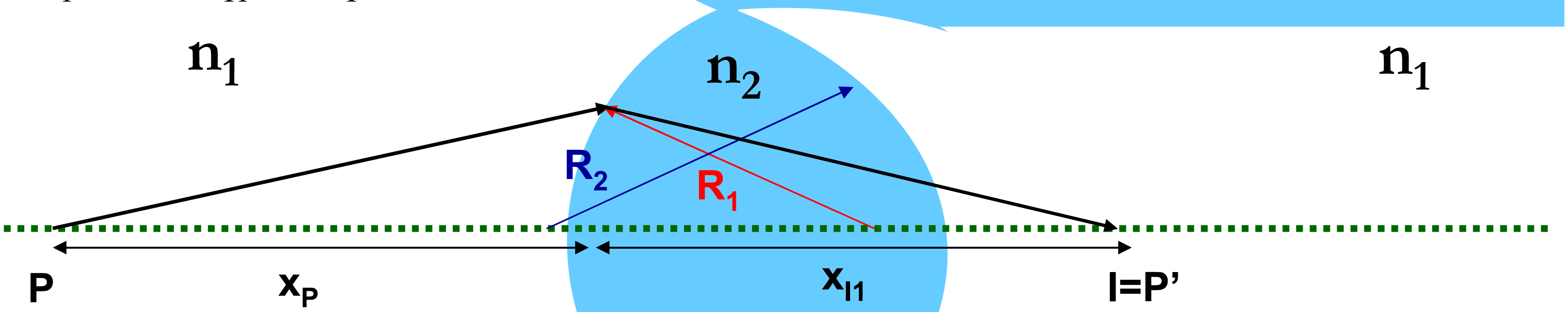
$$\text{Snell's law} \Rightarrow \theta_2 = \frac{n_1}{n_2}\theta_1 \Rightarrow \qquad \frac{y}{R} - \frac{y}{x_I} = \frac{n_1}{n_2}\left(\frac{y}{R} + \frac{y}{x_P}\right)$$

$$\boxed{\frac{n_2 - n_1}{R} = \frac{n_2}{x_I} + \frac{n_1}{x_P}}$$

# Thin Lenses: Starting




$$\frac{n_2 - n_1}{R_1} = \frac{n_2}{x_{I1}} + \frac{n_1}{x_P}$$




**P'** is the "new" object $\mathbf{x_{P'}=-x_{I1}}$ and we are going from $\mathbf{n_2}$ to $\mathbf{n_1}$ through a surface having $\mathbf{R_2}$ …

# Thin Lenses Equation

$$\frac{n_1 - n_2}{R_2} = \frac{n_1}{x_{IF}} + \frac{n_2}{x_{P'}} \quad \text{with} \quad \frac{1}{x_{P'}} = -\frac{1}{x_{I1}} = -\frac{n_2 - n_1}{n_2 R_1} + \frac{n_1}{n_2 x_P}$$

$$\boxed{\frac{1}{x_{IF}} + \frac{1}{x_P} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Thin Lenses Equation

$$\boxed{\frac{1}{f} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

**The Lens Maker's Formula**

$$\boxed{\frac{1}{x_{IF}} + \frac{1}{x_P} = \frac{1}{f}}$$

**Thin Lenses Equation**

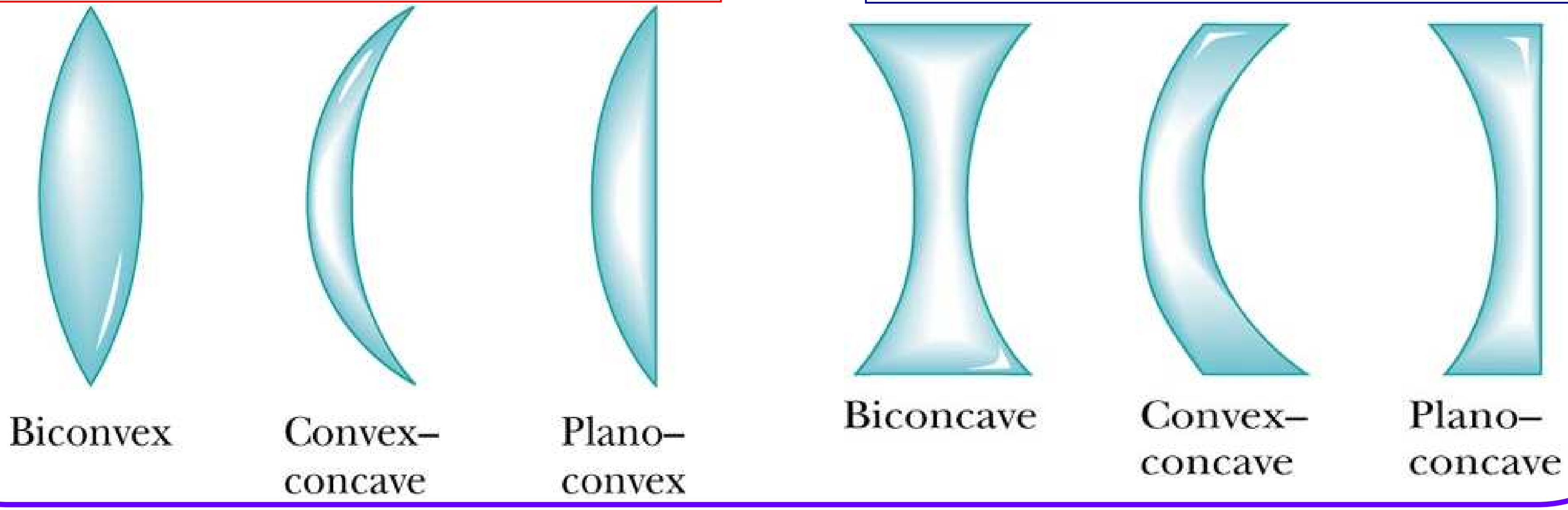
$\frac{1}{f}$ =(Converging) **Power** of the lens (in diopters, $m^{-1}$)

# Thin Lenses

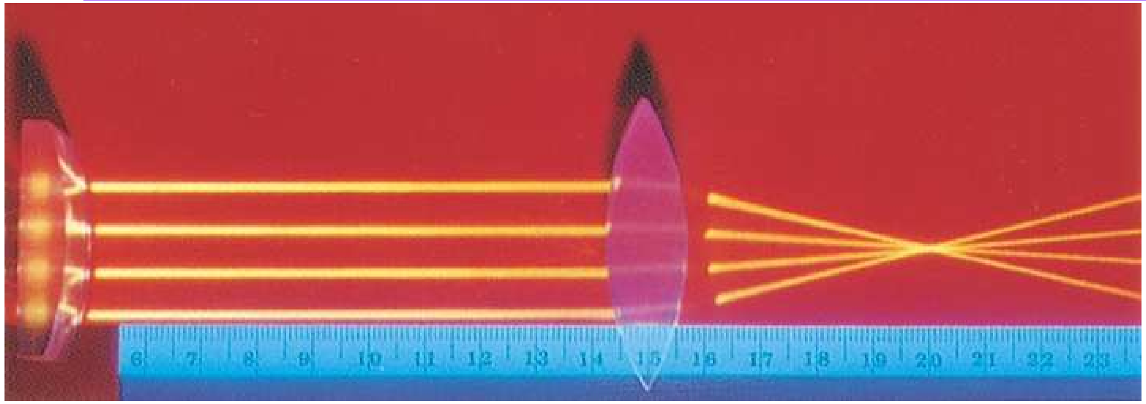
A **thin lens** consists of a piece of glass or plastic, ground so that each of its two refracting surfaces is a segment of either a sphere or a plane (a sphere of infinite radius). A lens is **thin** when the radii of curvature are much bigger than its thickness.

**Converging** lenses are thickest in the middle and have positive focal lengths

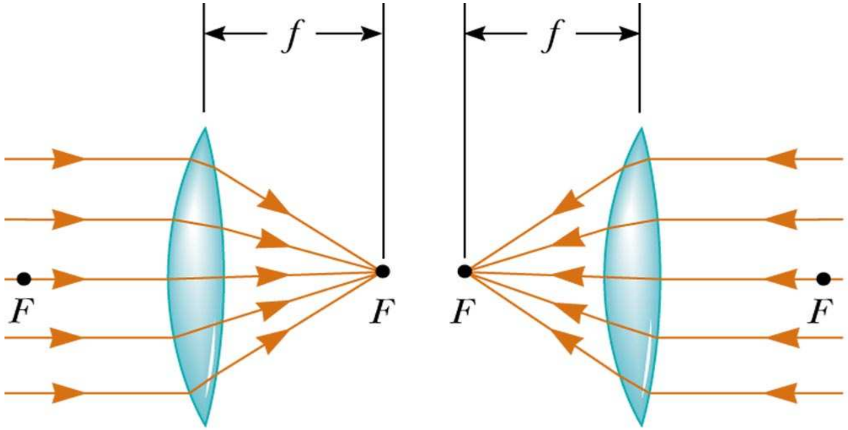
**Diverging** lenses are thickest at the edges and have negative focal lengths



# Converging Lens and Its Focal Length, *f*



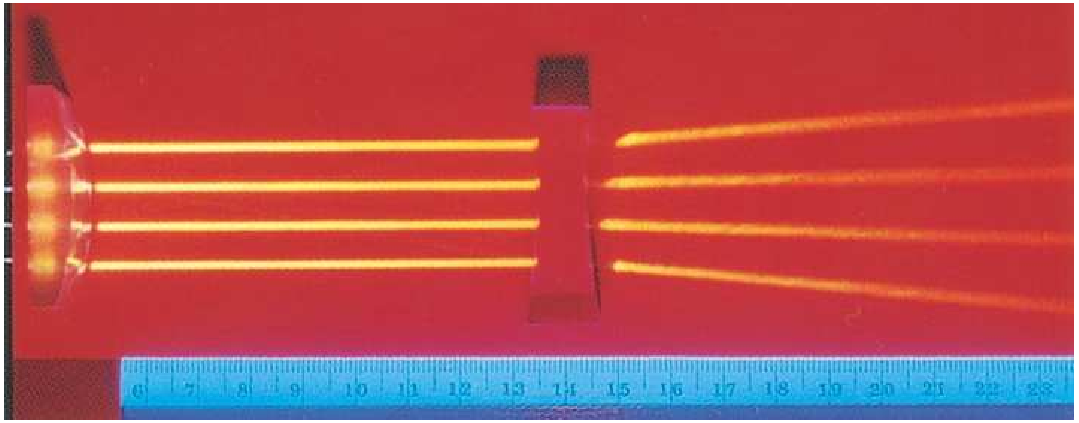

The parallel rays pass through the lens and converge at the focal point.



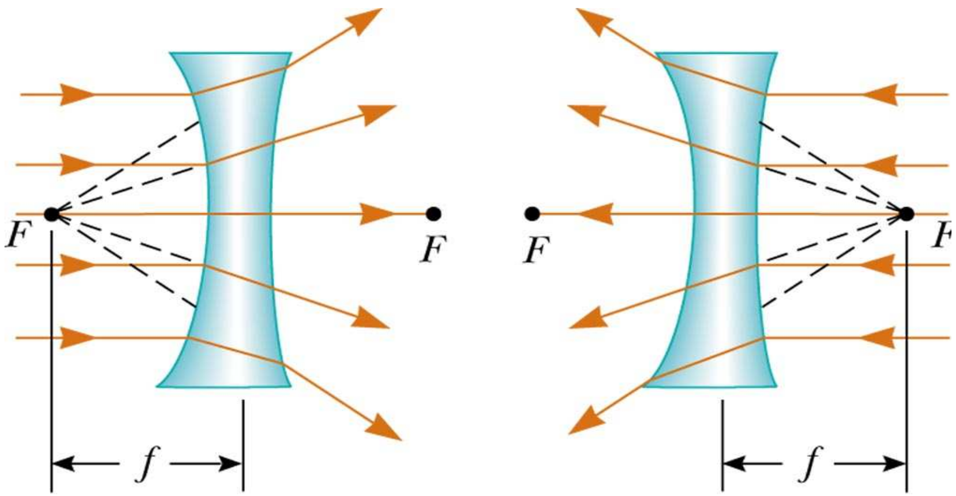

The **parallel rays** can come from the **left** or **right** of the lens. A thin lens has **two focal points** equidistance from the lens, corresponding to parallel rays from the left and from the right.

A **thin lens** is one in which the **distance between the surface of the lens and the center of the lens** is negligible compared to *f*.

# Diverging Lens and Its Focal Length, $f$

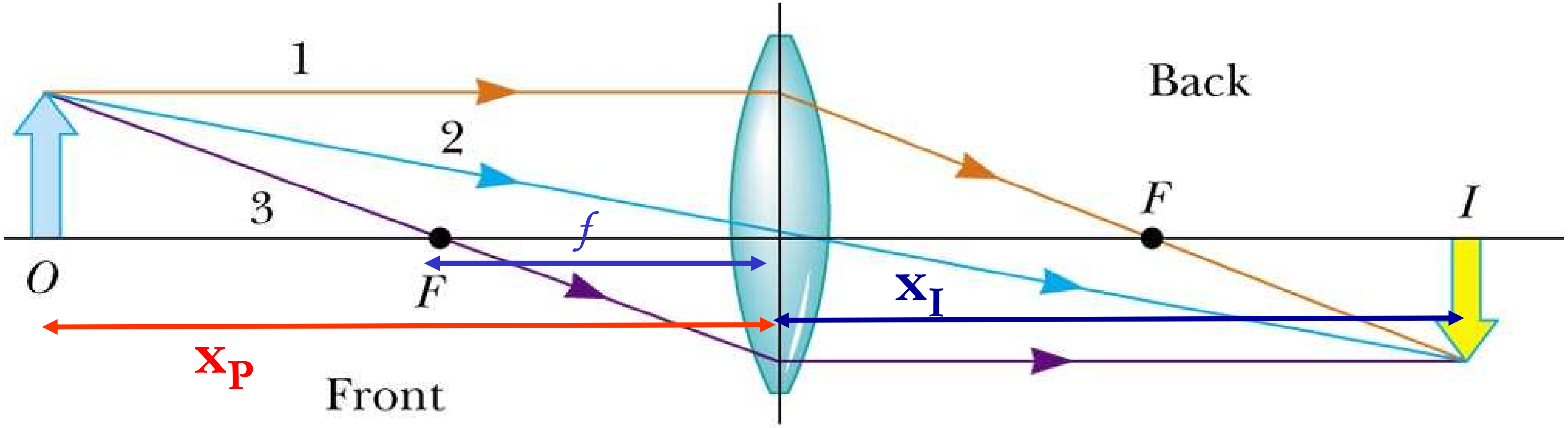



The parallel rays diverge after passing through the diverging lens.




The **focal point** is the point where the rays appear to have originated.

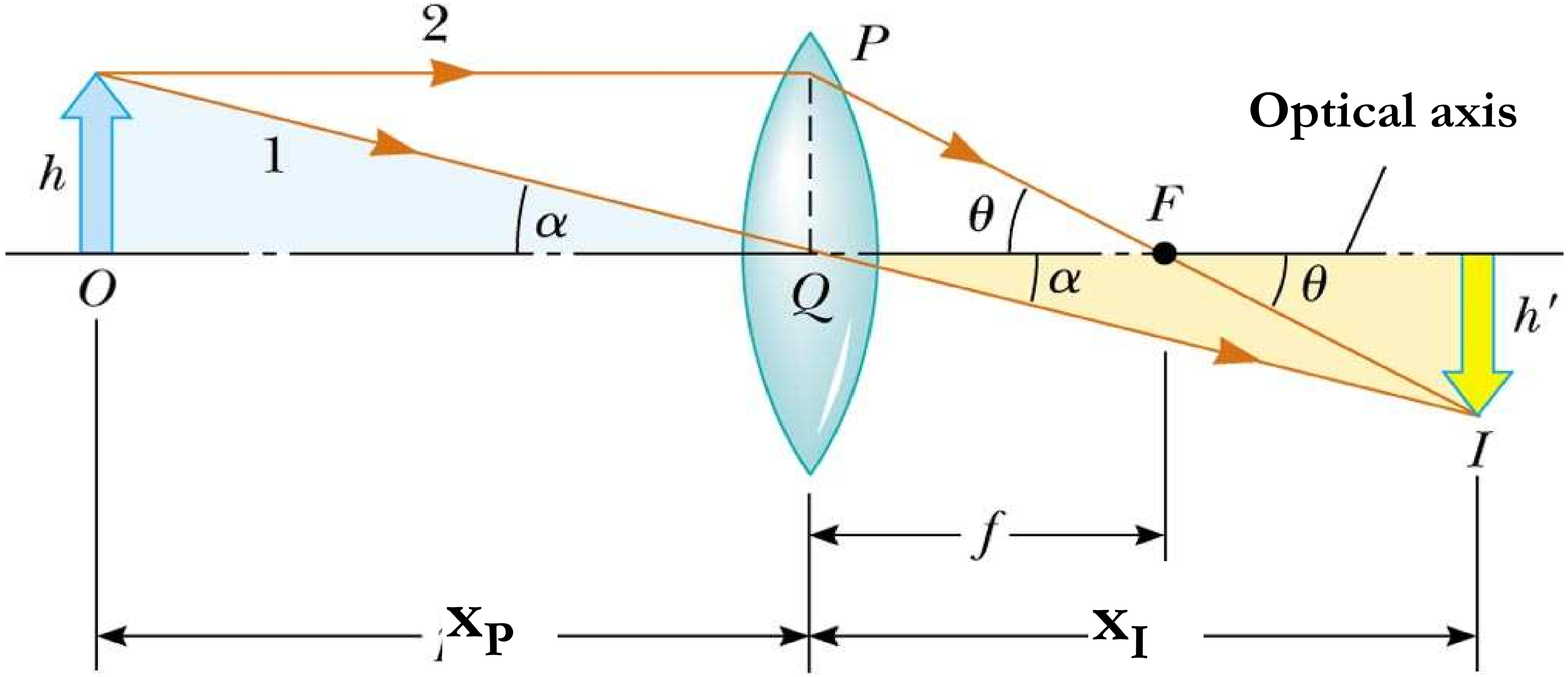
# Imaging




Three **principal rays** can help you to locate the image:

1. A ray parallel to the axis of the lens on the front side passes through the focal point on the back side.
2. A ray that passes through the center of the lens and does not get refracted.
3. A ray that passes through the focal point on the front side emerges parallel to the axis of the lens on the back side.

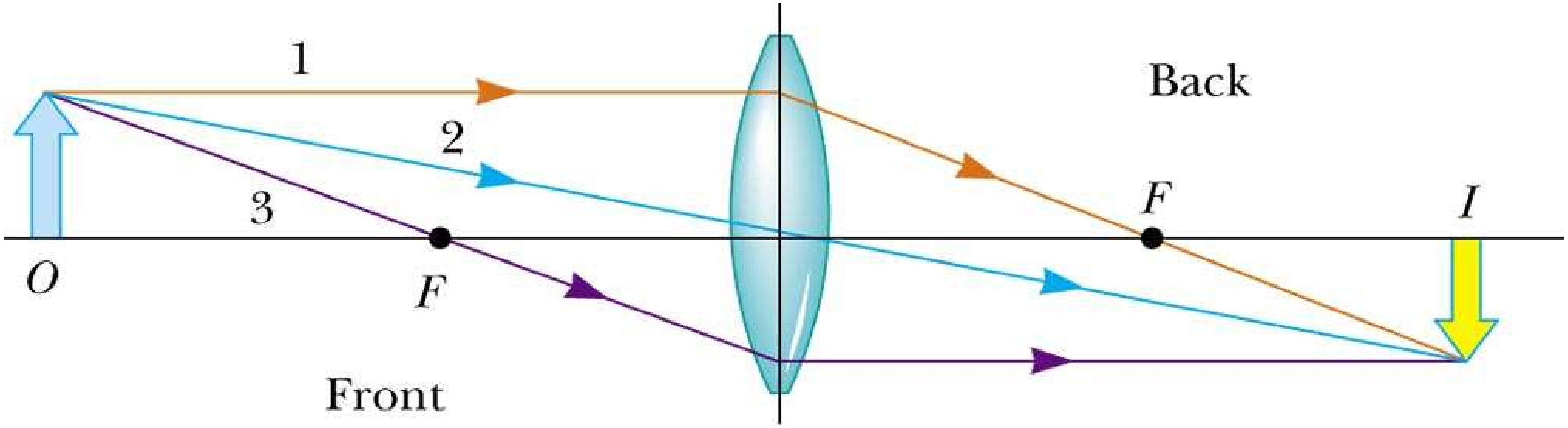
# Lens magnification




The two shaded triangles, blue and gold, are right-angled and similar. Therefore, for the lens magnification, **M**, we have:

$$\boxed{M = \frac{h'}{h} = -\frac{x_I}{x_P}}$$

# Ray Diagram for Converging Lens -1

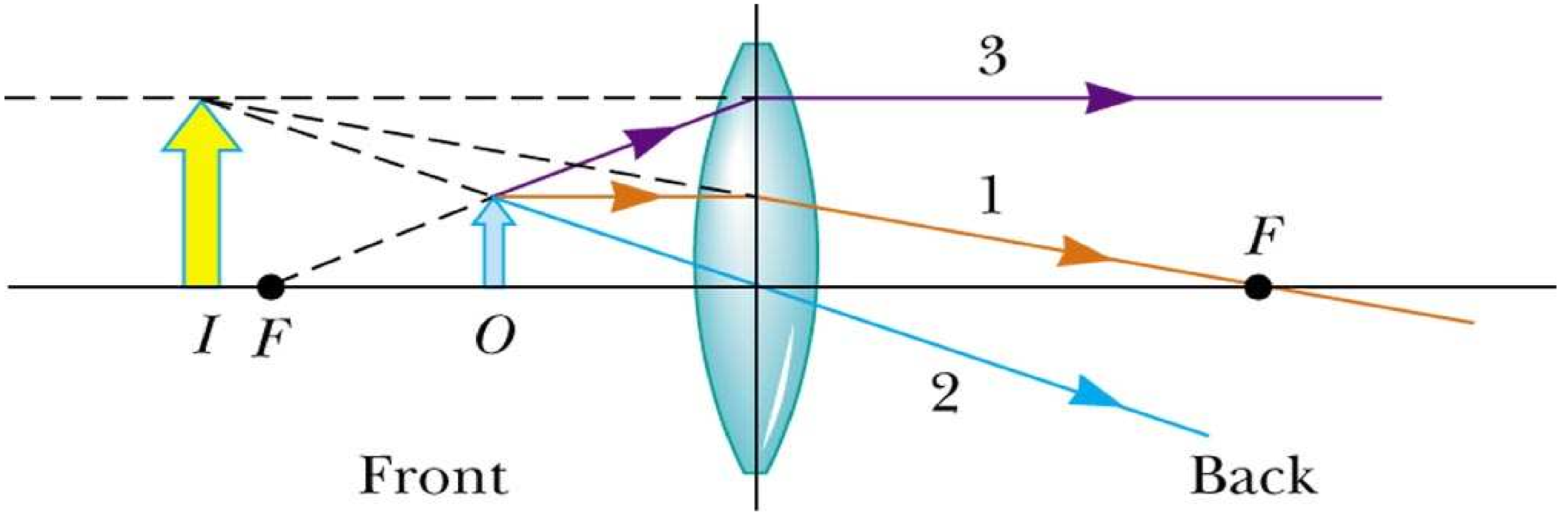



If the object is further away from the lens than the front focal point, $x_P > f$, $\Rightarrow$ the image is real, inverted, and located behind the lens

If the object is located at more than **2*f***, $x_P > 2f$, $\Rightarrow$ the image is reduced. If the object is located at $f < x_P < 2f$, the image is magnified. WHY?

$$\boxed{\frac{1}{x_I} + \frac{1}{x_P} = \frac{1}{f} \Rightarrow x_I = \frac{x_P f}{x_P - f}}$$

# Ray Diagram for Converging Lens -2



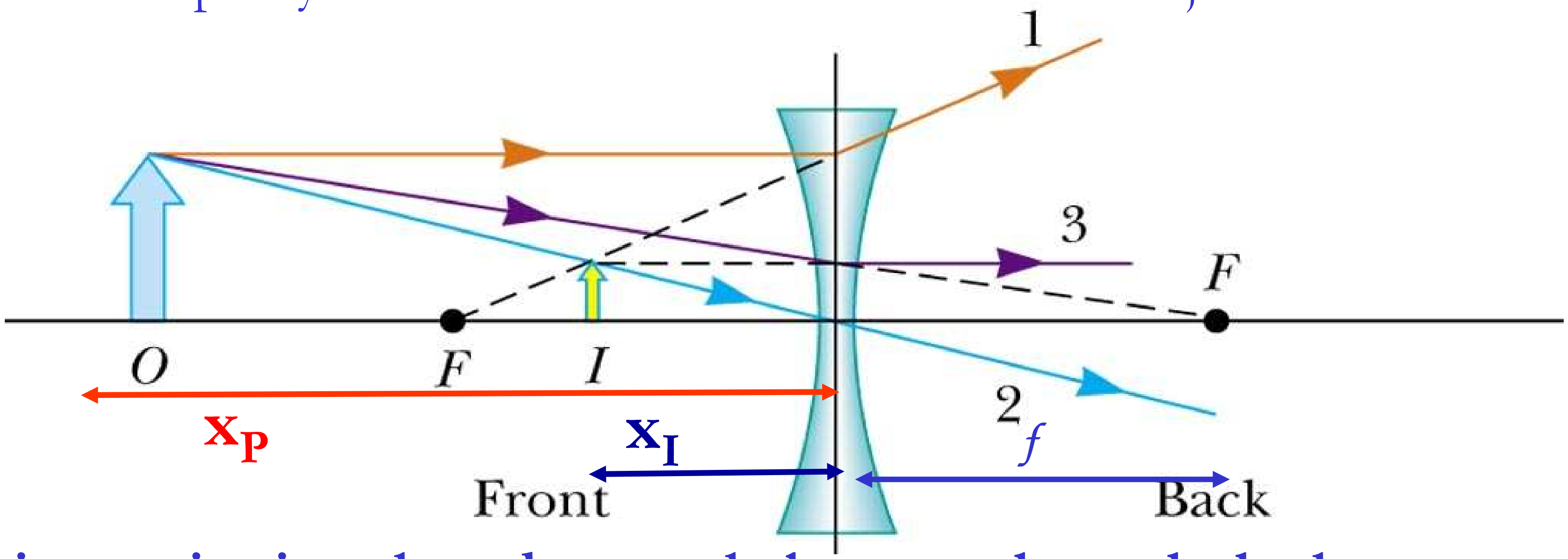

If the object is closer to the lens than the front focal point, $x_P < f$, $\Rightarrow$ the image is virtual, upright, magnified, and in front of the lens

$$\boxed{x_I = \frac{x_P f}{x_P - f}} \qquad \boxed{M = \frac{h'}{h} = -\frac{x_I}{x_P}}$$

The image can only be viewed from behind the lens!

# Ray Diagram for Diverging Lens

The situation is pretty much the same no matter where the object is located.




**The image is virtual, and can only be seen through the lens.**

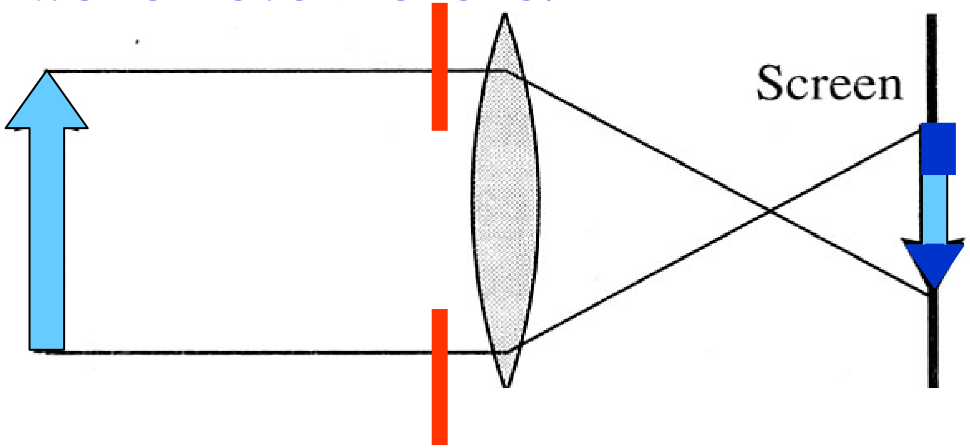
The image is upright and is always reduced in size compared with the object.

$$\boxed{x_I = \frac{x_P f}{x_P - f}}$$

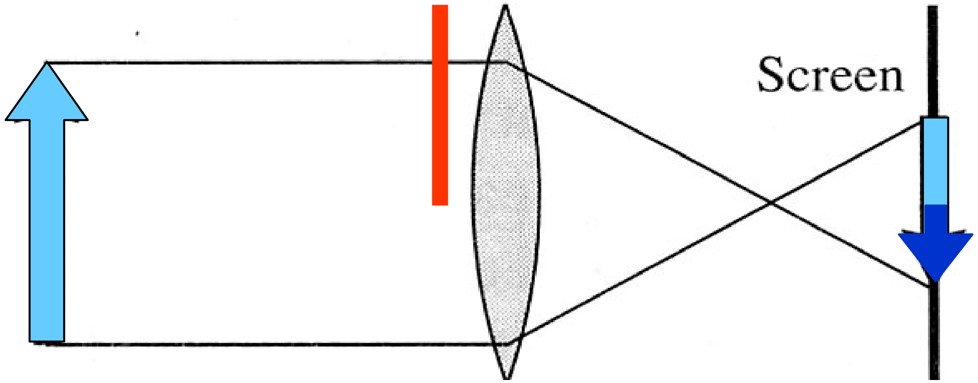
$$\boxed{M = \frac{h'}{h} = -\frac{x_I}{x_P}}$$

# Lens and Apertures

**What happens to the image if we put in an aperture?**
**If we remove the lens?**

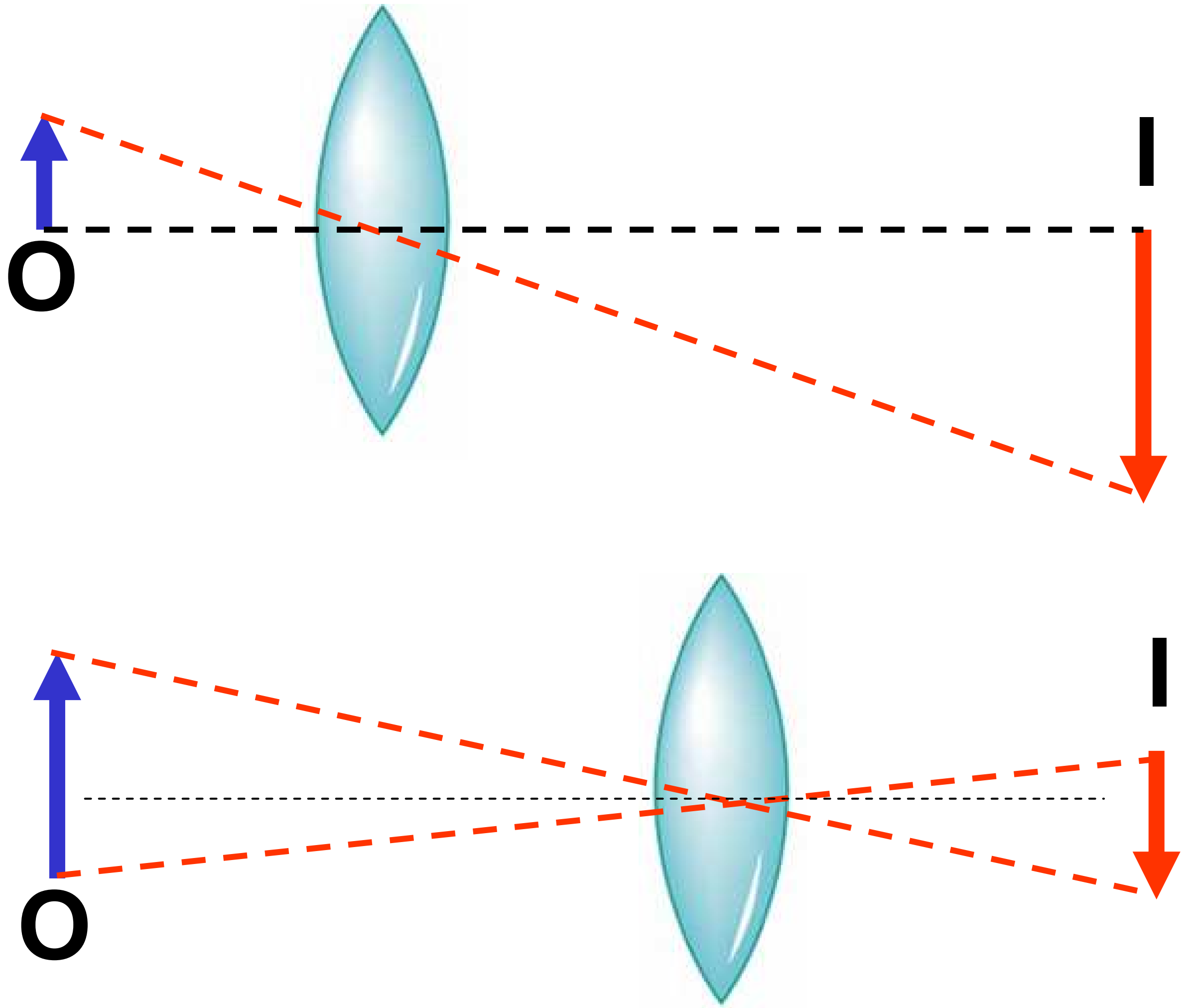



Top and bottom of image is less bright.

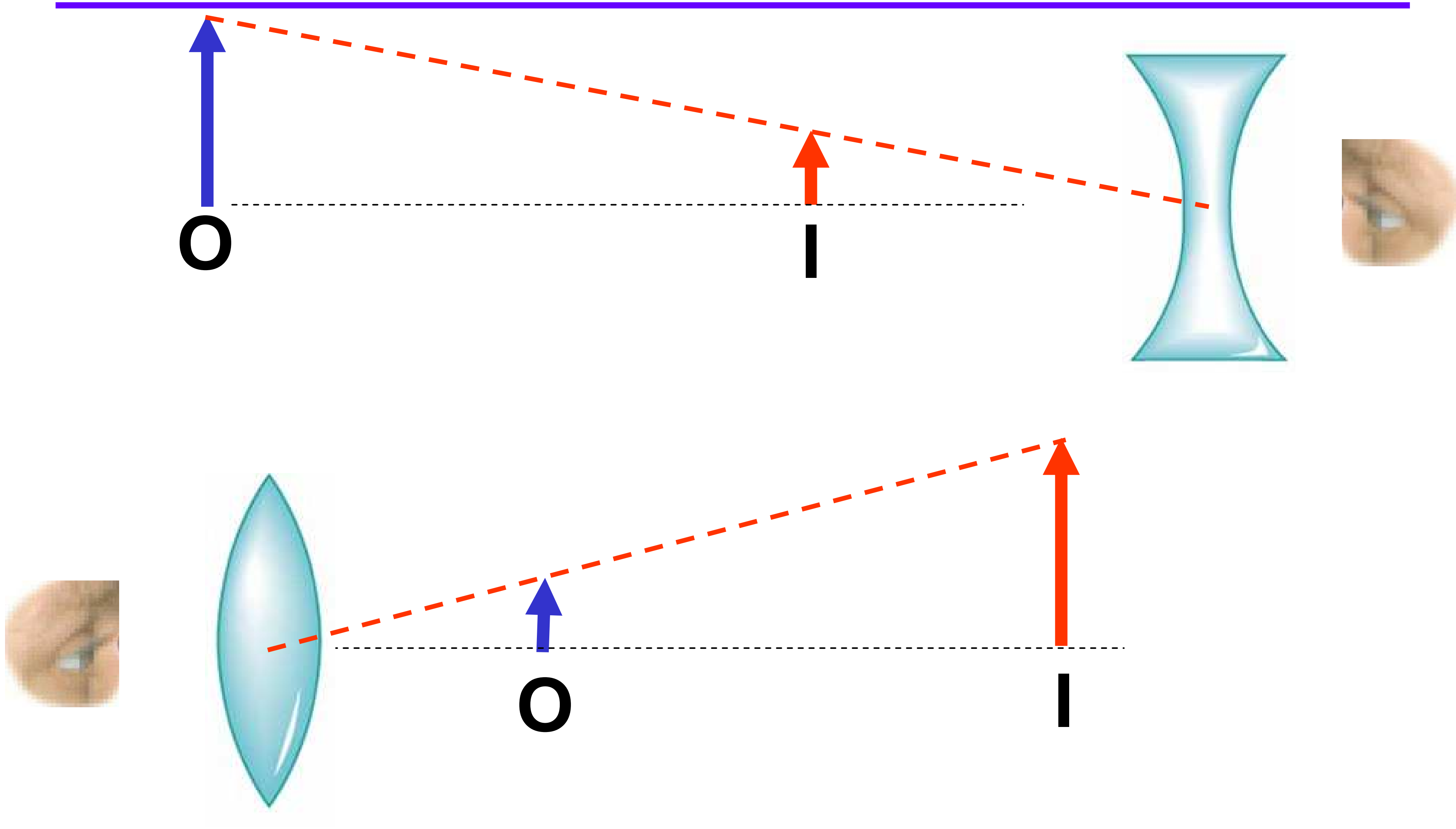



Bottom half of image is less bright

# Who has seen the lens?!



# Who has seen the lens?! -2



# Image Formation with a Lens -1

A light bulb is 56cm from a convex lens. The image appears on a screen 31cm on the other side of the lens.

(a) What is the focal length of the lens?

$$\frac{1}{f} = \frac{1}{\ell} + \frac{1}{\ell'} \rightarrow \frac{1}{f} = \frac{1}{56} + \frac{1}{31} = \frac{87}{56(31)}$$

$$f = 20cm$$

(b) What is the magnification factor of the image?

$$M = -\frac{\ell'}{\ell} = -\frac{31}{56} = -.55$$

Is the image inverted? YES!

# Image Formation with a Lens -2

A lens has a focal length of *35cm*.

(a) Find the type and height of the image when a *2.2cm* high object is placed *f+10cm* from the lens.

$$\frac{1}{f} = \frac{1}{\ell} + \frac{1}{\ell'} \rightarrow \frac{1}{\ell'} = \frac{1}{f} - \frac{1}{\ell}$$

$$\ell' = \frac{f\ell}{\ell - f} = \frac{f(f+10)}{10} = \frac{35(45)}{10} = 157.5cm$$

$$M = -\frac{\ell'}{\ell} = -\frac{35(45)}{10(45)} = -3.5 \rightarrow h' = -7.7cm$$

(b) Describe the image you got

**The image is real, behind the lens, and inverted.**

# Image Formation with a Lens -3

A lens has a focal length of *35cm*.

(a) Find the position and height of the image when a *2.2cm* high object is placed *f-10cm* from the lens.

$$\frac{1}{f} = \frac{1}{\ell} + \frac{1}{\ell'} \rightarrow \frac{1}{\ell'} = \frac{1}{f} - \frac{1}{\ell}$$

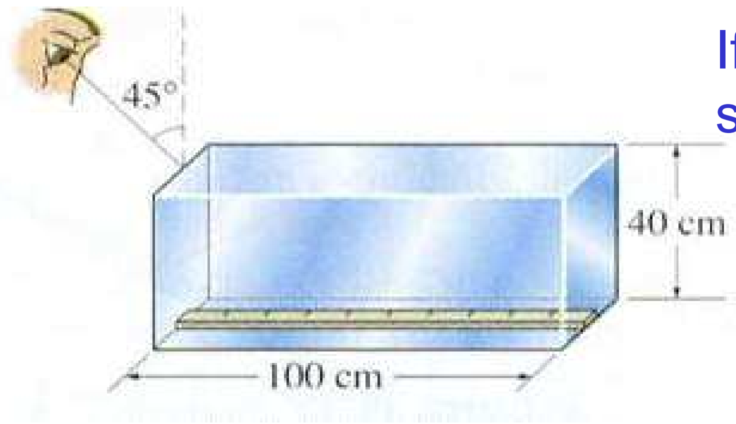
$$\ell' = \frac{f\ell}{\ell - f} = \frac{f(f-10)}{-10} = -\frac{35(25)}{10} = -87.5cm$$

$$M = -\frac{\ell'}{\ell} = \frac{35(25)}{10(25)} = 3.5 \rightarrow h' = 7.7cm$$

(b) Describe the image you got

**The image is virtual, in front of the lens, and upright.**

# Home Work -1




If you look into the corner of the cube at 45° (as shown) what mark on the meter stick do you see if

(a) The tank is empty?

(b) The tank is half full of water?

(c) The tank is full of water?

# Home Work -2

When light is propagating in glass with $n=1.52$, what is the critical angle when the glass is immersed in

(a) water ($n=1.33$)?

(b) in benzene ($n=1.50$)?

(c) In diiodomethane ($n=1.738$)?

# Home Work -3

A candle is 36cm from a concave mirror with a focal length 15cm.

(a) Where is the image?

(b) What is the magnification?

(c) Is the image real or virtual, upright or inverted?

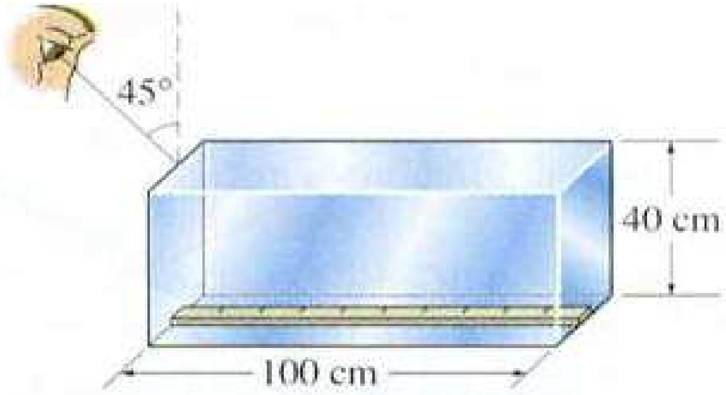
# Home Work -4

The image of an object in a *27cm* focal length concave mirror is upright and magnified by a factor of *3*. Where is the object?

# Home Work -5

You have a magnifying lens has a focal length of *30cm*. How far from the page should you hold the lens in order to see the print enlarged 3X?

# Water Tank




If you look into the corner of the cube at 45° (as shown) what mark on the meter stick do you see if

(a) The tank is empty? 40cm doh!

(b) The tank is half full of water?

$$\sin 45^\circ = 1.33 \sin\theta \rightarrow \theta = .56 rad = 32^\circ$$

$$x = 20 + 20\tan\theta = 32.5 cm$$

(c) The tank is full of water?

$$\sin 45^\circ = 1.33 \sin\theta \rightarrow \theta = .56 rad = 32^\circ$$

$$x = 40\tan\theta = 25 cm$$

# Critical Angle

When light is propagating in glass with *n=1.52*, what is the critical angle when the glass is immersed in (a) water (*n=1.33*)?

$$\text{(a) } n_1 \sin\theta_c = n_2 \sin(\theta = 90^\circ) = n_2$$

$$1.52 \sin\theta_c = 1.33 \rightarrow \theta_c = \sin^{-1}\frac{1.33}{1.52} = 61^\circ$$

(b) Benzene (*n=1.50*)?

$$\text{(b) } n_1 \sin\theta_c = n_2 \sin(\theta = 90^\circ) = n_2$$

$$1.52 \sin\theta_c = 1.50 \rightarrow \theta_c = \sin^{-1}\frac{1.50}{1.52} = 80.7^\circ$$

(c) diiodomethane (*n=1.738*)?

$$\text{(c) } n_1 \sin\theta_c = n_2 \sin(\theta = 90^\circ) = n_2$$

$$1.52 \sin\theta_c = 1.738 \rightarrow \sin\theta_c = \frac{1.738}{1.52} > 1 \text{ (trick question!)}$$

# Image Formation with a Mirror

A candle is 36cm from a concave mirror with a focal length 15cm.

(a) Where is the image?

$$\frac{1}{f} = \frac{1}{\ell} + \frac{1}{\ell'} \rightarrow \frac{1}{\ell'} = \frac{1}{f} - \frac{1}{\ell}$$

$$\ell' = \frac{f\ell}{\ell - f} = \frac{36(15)}{36 - 15} = 25.7cm$$

(b) What is the magnification?

$$M = \frac{h'}{h} = -\frac{\ell'}{\ell} = -\frac{36(21)}{36(15)} = -1.4$$

(c) Is the image real or virtual, upright or inverted?

**The image is in front of the mirror, hence real.**
**The magnification is negative, hence inverted.**

# Image Formation with a Mirror-2

The image of an object in a *27cm* focal length concave mirror is upright and magnified by a factor of *3*. Where is the object?

From the magnification factor of 3 we know that:

$$M = \frac{h'}{h} = -\frac{\ell'}{\ell} = 3 \rightarrow \ell' = -3\ell$$

From the lens equation:

$$\frac{1}{f} = \frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{\ell} - \frac{1}{3\ell} = \frac{2}{3\ell}$$

$$\ell = \frac{2}{3}f = 18cm$$

The image is *18cm* front of the mirror which is inside the focal point!

# Image Formation with a Lens -4

You have a magnifying lens has a focal length of *30cm*. How far from the page should you hold the lens in order to see the print enlarged 3X?


First note that the image is on the same side of the lens as the object, the page! This means that the image distance is negative. Hence:

$$M = \frac{h'}{h} = -\frac{\ell'}{\ell} = 3 \rightarrow \ell' = -3\ell$$

$$\frac{1}{f} = \frac{1}{\ell} + \frac{1}{\ell'} \rightarrow \frac{1}{30} = \frac{1}{\ell} - \frac{1}{3\ell} = \frac{2}{3\ell}$$

$$\ell = \frac{2}{3}30 = 20cm$$

This result means that the lens is *20cm* above the page and the image is *40cm* below the page, and the image is upright.

# Thin Lenses -2

# Title Here

- Beamsplitters: split the incident light into a reflected beam and a transmitted beam